Asymmetric Volatility in Commodity Markets

Yu-Fu Chen[a], Xiaoyi Mu[b,*]

a. Economic Studies, School of Business, University of Dundee, Dundee, DD1 4HN, UK. Email: y.f.chen@dundee.ac.uk
b. Centre for Energy, Petroleum and Mineral Law and Policy, University of Dundee, Dundee, DD1 4HN, UK. Email: x.mu@dundee.ac.uk
*. Corresponding author.

Abstract

This paper examines the relationship between return volatility and the level of returns in commodity markets. We develop a simple commodity price model and show that the volatility of price changes can be positively or negatively related to demand shocks depending on the demand and supply elasticities. We empirically examine the behaviour of volatility using both time-series conditional volatility models and historical volatility measures for a range of commodities including agricultural products, energy, industrial metals and precious metals. An "inverse leverage effect" – the conditional volatility is higher following a positive shock -- is found in more than half of the daily spot prices in time-series models. The effect is much weaker in 3-month futures market and monthly historical volatility measures. Only crude oil is found to exhibit a "leverage effect" – a higher volatility follows a negative shock, and the reason is explored in the context of its special market structure.

Keywords: Asymmetric volatility; commodity; inventory effect.

JEL Classification: G13, Q02, Q4

1. Introduction

This paper studies the relationship between return volatility and the level of returns in commodity markets. Specifically, we examine whether, and why, volatility responds asymmetrically to positive and negative return shocks in a range of commodities including agricultural products, energy, industrial metals and precious metals.

It is well-established in the finance literature that volatility in stock markets is asymmetric in that return volatility tends to be negatively correlated with stock returns. This is usually referred to as the "leverage effect" because as the stock price decreases firms' leverage (debt to equity ratio) naturally rises, which makes the stock riskier and increases its volatility (see Black, 1976 and Christie, 1982 for example). Other arguments include risk premium and herding behaviour.[1] Empirically, the leverage effect has found support in numerous studies employing both asymmetric GARCH models and implied volatility measures derived from option prices. Ederington and Guan (2010) provide a summary of the empirical literature.

Compared to that in equity markets, the return-volatility relationship for commodities is less well-studied and papers examining commodity price volatilities tend to focus on one or a group of specific commodities. Some noticeable empirical studies include Shawky *et al*. (2003), Hadsell *et al*. (2004), and Knittel and Roberts (2005) on electricity prices in deregulated US markets; Lee and Zyren (2007) on gasoline and heating oil; Hammoudeh and Yuan (2008) for gold, silver and copper; and Lucey and Tully (2006) for gold and silver; and Baur (2012) for gold. Except Hammoudeh and Yuan (2008), who reported that a negative return shock has a larger

[1] Influential studies in this area includes Schwert(1989), Campbell & Hentschel(1992), Bekaert and Wu (2000) and Wu(2001), to name a few.

impact on conditional volatility (i.e. leverage effect) of three-month copper futures in an EGARCH model using data for January 2, 1990 – May 1, 2006, all other aforementioned studies find that a positive return shock has a larger impact on conditional volatility (i.e. inverse leverage effect) in their respectively studied commodities. More recently, Chiarella et al. (2016) and Bauer and Dimpfl (2018) find that a negative return shock has, in general, a larger impact on volatility for crude oil. Thus, the commodity return-volatility relation could be positive or negative depending on the specific commodity and sample period.

A positive return-volatility relationship is often explained by the theory of storage (Working, 1949; Litzenberger and Rabinowitz, 1995; Routledge et al., 2000). When inventory is low, the risk of supply shortage is high, leading to an increase in prices and volatility. Conversely, when inventory is high, the risk of supply shortage is low, resulting in a fall in prices and volatility. However, it is difficult to explain the negative return-volatility relationship found in crude oil and some other commodities with the theory of storage[2]. Hence a new theoretical framework is needed to understand the differences in asymmetric volatility effect across commodities.

We complement to the literature in several important ways. First, we develop a commodity pricing model where the demand is stochastically fluctuating and a representative firm maximizes its profit. Under fairly general assumptions, we show that the volatility of price changes can be positively or negatively related to demand

[2] Chiarella et al. (2016) distinguish between investment commodities and consumption commodities. Based on an empirical analysis of gold and crude oil, they argue that normal consumption goods, such as crude oil, are characterized by negative return-volatility relationship (leverage effect) and investment goods, such as gold, are characterized by positive return-volatility relationship (inverse leverage effect) particularly during more volatile periods. However, this generalization is misleading. As we shall show below, the positive return-volatility relationship is found in a variety of commodities including soybean, wheat, energy products, and industrial metals, most of which can't be classified as investment commodities.

shocks depending on the demand and supply elasticities. The variance of demand shocks is constant in our model, but the curvature of demand and supply relationship result in different association between the price volatility and the state of demand. When the cost curve of the representative firm is convex, as one would expect for most commodities in spot markets, the volatility of price is higher when the demand is high, leading to an "inverse leverage effect". Conversely, if the cost curve is concave, the price volatility is negatively related to the state of the demand. In such cases, a "leverage effect" would emerge.[3] In the literature, we are aware of only two theoretical studies that explored the differential effect of positive/negative shocks on price volatility in commodity markets. Under a rational expectations competitive storage framework, Deaton and Laroque (1992) show that the conditional variance of price is an increasing function of current price levels if the supply curve is convex, indicating a higher volatility following a positive demand shock. Carlson *et al.* (2007) develop a model of equilibrium prices for exhaustible resources and demonstrate that the stochastic volatility of prices could be negatively or positively related to demand shocks. In comparison, our model is more general in that it neither relies on inventory nor is limited to exhaustible resources.[4] In fact, our model is also flexible to specifications of market structure.

Second, we empirically examine the return-volatility relationship in 19 commodities including three agricultural products, six energy products, six industrial metals and four precious metals, using both time-series models and monthly historical volatility measures. In time-series models, asymmetric volatility were found to be

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[3] Although the asymmetric return-volatility relationship in commodity markets does not arise from financial leverage, following the literature we continue to refer it as to leverage (or inverse leverage) effect.

[4] Indeed, as will be shown in the empirical section, asymmetric volatility presents for both exhaustible and non-exhaustible commodities such as agricultural products, and storable as well as non-storable goods (electricity).

statistically significant in 12 out of the 19 daily spot prices. The result is robust when discontinuities were accounted for. The majority of commodities show a significant positive relationship between prices and conditional volatility – the conditional volatility tends to be higher following a positive demand shock, i.e. “inverse leverage effect” – in the spot market. Crude oil is different and exhibit a negative relationship between price and volatility – the volatility tends to be higher after a negative demand shock, i.e. “leverage effect”. Consistent with our model predictions, the positive relationship between prices and volatility is strongest in commodities that are hard to store (for example, electricity) or supplied from more geographically constrained markets, and is weaker in the price of third-month futures and monthly historical volatility measures.

The somewhat surprising result for crude oil could be consistent with our model prediction in that the supply relationship for crude oil may actually appear concave because of the unique market and cost structures of crude oil. Crude oil is the only commodity whose price is strongly influenced by an organization with significant market power, i.e. OPEC.[5] The combination of low marginal cost among major oil producers and the possibility of OPEC to limit production in times of negative demand shocks makes the supply relationship to appear concave.

We are not the first to study the asymmetric volatility effect across commodities. In the literature, the paper that is closest to ours is Bauer and Dimpfl (2018) who study a similar set of commodities and compare those to equity markets. In comparison, we not only provide a theoretical framework, which is more general than the theory of storage, for understanding the differences in the return-volatility relationship in

[5] Although there is a debate on which model best describes the role of OPEC in global oil market, there is a consensus in the literature that OPEC has market power to influence the oil price.

commodity markets, but also empirically focus on the comparison between commodities, and documenting how the asymmetric volatility effect change with time-to-maturity.

The rest of the paper is structured as follows. In the next section, we outline the theoretical framework. Section 3 provides a description of the data. The estimation results of the asymmetric GARCH models, using daily returns are reported in Section 4. Section 5 reports an OLS estimation for monthly historical volatility measures and Section 6 concludes.

2. A Theoretical Framework of Commodity Price Volatility

In this section we outline a commodity pricing framework in which the commodity price is assumed to be determined by market demand and supply forces and the volatility is driven by shocks in demand. We explore the relationship between price (return) and volatility in commodity markets. The model is in similar spirit to the work of Carlson *et al* (2007). However, our model is not constrained to exhaustible resources. Following the finance literature, we refer to the negative association between price (return) and volatility as the leverage effect; and the positive association, i.e., the volatility is higher when there are positive demand shocks, as the inverse leverage effect even though the asymmetries in commodity markets do not arise from financial leverage.

We start from a representative firm's profit maximisation decision when it faces an uncertain demand curve and an upward sloping supply curve.[6] The inverse demand function is given by

[6] An alternative strategy is to model the aggregate demand and supply of the market, which would require assumptions about market structure. In our model, the market structure can be incorporated in the demand function.

$$p_t = Z_t q_t^{\frac{1-\psi}{\psi}}, \psi \geq 1, \tag{1}$$

where p denotes real prices faced by the firm, q is the output of the firm, ψ is an elasticity parameter and $\psi = 1$ when the demand curve is flat. The larger the ψ, the steeper is the demand curve. Z_t follows a continuous stochastic process, to be specified later.

The firm maximise its profits,

$$\max_q \quad Z_t q_t^{\frac{1}{\psi}} - c(Z_t) q_t^{\gamma}, \tag{2}$$

where $c(Z_t)$ is the adjustment cost function relating to demand shocks, parameter γ denotes whether the cost function is convex or concave supply. The supply relationship is convex if $\gamma > 1$ and concave if $0 < \gamma < 1$. To make it tractable, we further assume that $c(Z_t)$ is linear in Z_t: $c(Z_t) = c_0(1 + \epsilon Z_t)$, where c_0 represents the fixed cost and ϵ is the sensitivity coefficient related to demand shocks and $\epsilon \geq 0$. If the firm's supply curve can accommodate demand shocks without incurring additional adjustment cost, then $\epsilon = 0$. However, in most cases, we envisage some positive adjustment cost such that $\epsilon > 0$. The profit maximisation problem (2) can be rewritten as

$$\max_q \quad Z_t q_t^{\frac{1}{\psi}} - c_0(1 + \epsilon Z_t) q_t^{\gamma}, \tag{2a}$$

The optimal level of output requires $\frac{1}{\psi} Z_t q_t^{\frac{1}{\psi}-1} = \gamma c_0 (1 + \epsilon Z_t) q_t^{\gamma-1}$, and we then have the optimal output,

$$q_t = \left(\frac{Z_t}{\psi c_0 (1+\epsilon Z_t)\gamma}\right)^{\frac{1}{\left(\gamma - \frac{1}{\psi}\right)}}. \tag{3}$$

Substituting equation (3) back into the isoelastic demand function equation (1) gives

$$p_t = (\psi c_0 (1 + \epsilon Z_t)\gamma)^{-\frac{\frac{1-\psi}{\psi}}{\left(\gamma - \frac{1}{\psi}\right)}} Z_t^{1+\frac{\frac{1-\psi}{\psi}}{\left(\gamma - \frac{1}{\psi}\right)}}. \tag{4}$$

Assume the demand uncertainty parameter Z_t follows a standard geometrical Brownian motion,

$$dZ_t = \eta Z_t dt + \sigma Z_t dW_t, \tag{5}$$

where η is the demand drift parameter, σ is the volatility parameter and W is the standard Wiener process. In the appendix, we show that the result holds if Z_t follows a mean-reverting process. By applying Ito's Lemma to equation (4), we have

$$dp_t = \left(\left(1 + \frac{\alpha}{(1+\epsilon Z_t)}\right)\eta + \frac{\alpha\epsilon^2\sigma^2 Z_t^2}{2(1+\epsilon Z_t)^2} - \frac{1}{2}(1+\alpha)\sigma^2\right)p_t dt$$

$$+\left(1 + \frac{\alpha}{(1+\epsilon Z_t)}\right)\sigma p_t dW_t \tag{6}$$

where $\alpha = \frac{\frac{1-\psi}{\psi}}{\left(\gamma - \frac{1}{\psi}\right)}$.

Thus, let σ_p denote the volatility of price change:

$$\sigma_p = \left(1 + \frac{\alpha}{(1+\epsilon Z_t)}\right)\sigma. \tag{6a}$$

Since $\psi > 1$, the effect of Z_t on σ_p is determined by $\gamma - \frac{1}{\psi}$ and to a second degree, ϵ. If $\gamma > \frac{1}{\psi}$, then $\alpha < 0$ and σ_p is increasing in Z_t; and if $\gamma < \frac{1}{\psi}$, then $\alpha > 0$ and σ_p is decreasing in Z_t. The effect of ϵ is to amplify or dampen Z. If $\epsilon = 0$, σ_p is not affected by Z_t. The intuition follows if one realizes that $\frac{1}{\psi}$ and γ are respectively the elasticities of the firm's revenue and cost with respect to output. When the cost curve is steeper than the revenue curve, constant volatility changes in demand results in high volatile equilibrium price at high demand state. Conversely, if the cost curve is flatter than the revenue curve, then high volatile equilibrium price could occur in low demand state. If the adjustment cost is zero, then the price volatility is not affected by demand shocks.

Figure 1 presents the state dependence of the volatility under different parameterizations. When ψ is large (as likely in the case for spot market), α could be positive and the price volatility decreases in Z_t only if γ is extremely small →0 (i.e., the cost curve is extremely concave as shown in Figure 1b). In all other cases, α is negative and hence the price (return) volatility increases in Z_t, resulting an "inverse leverage" effect (Figure 1a). In fact, as long as $\gamma > 1$ (that is, when the supply relationship is convex), the price volatility is higher when there is a positive demand shock. We expect this to be the case for most commodities in spot markets.

When ψ is small (→1), the demand curve is nearly flat and α approaches zero. Unless $(\gamma \rightarrow 1) < 1$, the effect of Z_t on price volatility σ_p diminishes. As shown in Figure 1c, although there is a degree of state dependence, the change in volatility σ_p is less than 1 percent as log(Z) changes from -1 to 1. When $(\gamma \rightarrow 1) < \frac{1}{\psi} < 1$ (i.e., when the supply relationship is concave), α is negative and the price volatility σ_p decreases in Z_t, resulting a "leverage effect" as shown in Figure 1d.

Since the state dependence of volatility is determined by demand and supply elasticities, we have the following general conjectures when comparing different commodities across temporally and spatially different markets.

First, in spot markets where the demand parameter ψ is likely to be large (that is, the demand curve is relatively steep), the "inverse leverage effect" is more pronounced for commodities whose supply are more geographically constrained and/or whose storage is more costly. The reason is simple; if the supply of the commodity is geographically constrained, or it cannot be easily stored, then the cost of adjusting supply is higher, implying a higher γ. This is the case depicted in Figure 1a.

Second, for the same commodity, the "inverse leverage effect" is more pronounced in spot markets than in longer term forward or futures markets. As the time horizon lengthens, both the inverse demand curve and supply relationship will flatten, so that $\gamma < \frac{1}{\psi}$ is more likely to hold in the forward or futures markets than in the spot markets.

3. Data

To empirically examine the possible asymmetries in commodity price volatilities, we obtain daily price data of 19 commodities including 3 agricultural products, 6 energy products, 6 industrial metals and 4 precious metals. Except the data on energy products that were downloaded from the Energy Information Administration (EIA) of the US, the data for all other commodities were obtained from Datastream™,[7] which sourced data from the US Department of Agriculture and the London Metal Exchange, respectively. Except for electricity (NEPOOL and PJMW), the prices of other energy products are daily settlement prices at New York Mercantile Exchange. The electricity prices are day-ahead volume-weighted average prices from the New England Power Pool (NEPOOL) and Pennsylvania-New Jersey-Maryland West hub (PJMW). We have data on cash prices for agricultural products and precious metals, cash price and 3-month futures prices for industrial metals, front-month and 3-month futures prices for crude oil, gasoline, heating oil and natural gas.[8] The appendix gives a detailed data description. In what follows, we shall refer to all spot prices, cash prices, front-month futures prices and day-ahead prices for electricity as “spot prices”.

Since the price series are generally non-stationary in levels, we calculate daily returns which are defined as changes of the logarithm of the daily prices: $r_t = ln(P_t/P_{t-1})$. Table 1 presents the summary statistics of the 30 return series, including 19 calculated

[7] The Datastream has prices for other commodities such as cotton, coffee, lard and orange juice but the trading of these commodities are not liquid even as 2010 and therefore were excluded from this study.
[8] The front-month futures prices for energy products are highly correlated with the spot prices as implied by the cash and carry model. The futures prices are preferred because these prices are recorded in central exchanges whereas the spot prices are reported by various news agencies.

on the basis of spot prices, 10 on 3-month futures prices for energy and industrial metals, along with the return of a stock market index, S&P 500, for comparison. Several patterns in this table are noteworthy. First, with the exception of the two electricity price series, the mean returns for all other commodities are positive, indicating an overall increasing trend in commodity prices for the respective sample periods. Second, the standard deviation and the range of returns indicate the commodity prices are highly volatile. The standard deviations of all commodities but gold exceed that of the S&P500 index. It is not surprising that the electricity price has the highest volatility given the fact that it is non-storable and any shocks to demand and supply will be manifested in price. Third, comparing the standard deviations of the spot price returns with that of the 3-month futures of energy products and industrial metals, it is clear that the return of the spot prices is more volatile than that of the 3-month futures, which is consistent with the "Samuelson effect" in that the price volatility of futures contracts declines as the contract horizon lengthens. Fourth, the majority of returns exhibit some degree of negative skewness, although generally not as skewed as the S&P500 index. This is in contrast to some of earlier findings in the literature (Gorton, Hayashi, and Rouwenhorst (2007) and Deaton and Laroque (1992)) which reports positive skewness in commodity price returns. The negative skewness could be consistent with a situation where the market is well supplied and negative demand shocks cause price to fall more than price spikes either in frequency or magnitude. [9] Fifth, perhaps not surprisingly, the commodity returns are leptokurtic, although the kurtosis is lower than that of the S&P500 index. Leptokurtosis suggests a student *t*-distribution is appropriate in empirical modelling.

[9] According to Deaton and Laroque (1992), positive skewness in commodity prices can be explained by the theory of storage where the low level of inventory causes positive price spikes which exceed in magnitude the negative price spikes at times of high level of inventory.

4. Time-series Estimates of Daily Returns

4.1 Model Specification

We begin our empirical investigation by estimating two standard asymmetric conditional volatility GARCH models: the threshold GARCH (TGARCH, or GJR-GARCH) model of Glosten *et al*. (1993) and the exponential GARCH (EGARCH) model of Nelson (1991).[10] In the mean equation, we include the log return of S&P 500 index (*RSNP*) to control the opportunity cost of investing in the commodity market. Since the commodity prices are all quoted in US dollars, changes in the US dollar exchange rate could affect the prices of commodities. In particular, Chen, Rogoff and Rossi (2010) argue that the exchange rates of "commodity currencies" have remarkably robust power in predicting global commodity prices, but the reverse relationship is less robust. To control the exchange rate effect, we include a log return on the trade volume weighted US dollar index (*RUSD*) in the mean equation. Both the S&P500 index and the US dollar index were obtained from the Federal Reserve Bank of St Louis FRED™ database.

The mean equation takes the following form[11]:

$$r_t = \alpha_0 + \sum_{j=1}^{k} \beta_j r_{t-j} + \alpha_1 \sigma_t + \alpha_2 RSNP_t + \alpha_3 RUSD_t + e_t \tag{7}$$

[10] In what follows, we only report the results from the TGARCH model. The results from the EGARCH model are similar to those obtained from the TGARCH model. To save space, the results are not reported, but available upon request from the authors.

[11] Despite the potential endogeneity between commodity returns and stock returns, we include *RSNP* in the mean equation to see the correlation between them. We have also estimated specifications which exclude *RSNP*, *RUSD* and the GARCH-in-mean terms. The results are little changed from those reported below and are available from the authors upon request.

where r_t is the daily log return of a series and e_t is assumed to be normally distributed with mean zero and variance σ_t^2. The number of the lags in the mean equation is chosen on the basis of the Akaike information criterion (AIC).

The variance equation for the TGARCH model is

$$\sigma_t^2 = \gamma_0 + \gamma_1 e_{t-1}^2 + \gamma_2 e(-)_{t-1}^2 + \gamma_3 \sigma_{t-1}^2 \qquad (8)$$

where $e(-)_t = e_t$ if $e_t < 0$ and 0 if $e_t \geq 0$. Thus the contribution of a negative shock to the conditional variance is determined by the coefficient ($\gamma_1 + \gamma_2$) while the contribution of a positive shock depends solely on γ_1. If γ_2 is negative, then a positive return shock has a larger impact on the conditional variance than a negative shock. If $\gamma_2 = 0$, then the TGARCH model collapse to the standard GARCH(1,1) model and the return-volatility relationship is symmetric.

Recent research has highlighted the importance of accounting for jumps in modelling commodity prices. For example, Wilmot and Mason (2013) show that the combination of jumps with a time-varying volatility significantly improves the model fit for daily and weekly crude oil prices over other competing models (pure jump-diffusion or pure GARCH) in explaining. The idea for jumps is that the arrival of new information often leads to unexpectedly large changes in the underlying asset prices over a short period of time. Such events generate the relatively fat tails in distribution returns. Following this literature, we also estimate a jump-diffusion asymmetric GARCH model where the mean equation (7) is augmented by the jump process while the variance takes the form of equation (8). The jump-augmented mean equation is as follows:

$$r_t = \alpha_0 + \sum_{j=1}^{k} \beta_j r_{t-j} + \alpha_1 \sigma_t + \alpha_2 RSNP_t + \alpha_3 RUSD_t + J_t + e_t \qquad (9)$$

where $J_t = \sum_{i=0}^{n_t} Y_{t,i}$. The jump size, $Y_{t,i}$, is assumed to be independent and normally distributed with mean θ and variance δ^2. $n_t \in \{0, 1, 2, \cdots\}$ is the number of jumps that occur over the interval (*t-1, t*) and is governed by a Poisson distribution:

$$P_{(N_t=j)} = \frac{\exp(-\lambda)\lambda^j}{j!} \tag{10}$$

λ is the jump intensity parameter which describes the mean number of jumps occurring per unit of time. The parameters are estimated using maximum likelihood with asymptotically normal distributions.

4.2 Estimation Results

Estimations of the TGARCH models using spot prices are reported in Table 2 and Table 3. The asymmetric GARCH parameter γ_2 in Table 2 is negative and statistically significant at the five percent level for 11 commodities (highlighted in bold), indicating a negative return shock has a smaller impact on conditional volatility than a positive shock. Take the estimated coefficients γ_1 and γ_2 for soybean as an example, the estimates suggest that the impact of a positive price shock on the next day's volatility is, on average, more than double the impact of a negative shock of the same magnitude.

Looking at the results for each group of commodities, the pattern of the γ_2 parameter is broadly consistent with our first conjecture. For example, in the energy group, commodities that yield statistically significant and negative γ_2 coefficients (gasoline, heating oil, NEPOOL and PJMW) generally have a more geographically constrained market or are more costly to store than those having insignificant or positive γ_2 coefficients. On the face, it may appear surprising that the estimated γ_2 coefficients are negative and statistically significant for gasoline and heating oil, while positive and statistically significant for crude oil. In other words, while gasoline and heating oil appear to have an “inverse leverage effect”, crude oil exhibits a “leverage effect” similar to that observed in financial markets. Notwithstanding, the positive γ_2 coefficient for crude oil could be consistent with the generally-held view that crude oil is a fungible product which is supplied from a broader market[12] and there is large inventories in the United States, all of which make the demand curve for the representative firm and the supply relationship flatter. We defer a fuller account of the somewhat surprising result for crude oil in next session. In contrast, the markets for gasoline and heating oil are geographically constrained in the US and to some extent even fragmented within state boundaries due to varying product specifications.[13] The prices are therefore more susceptible to shocks to local supply and demand conditions. It is therefore conceivable that both the supply and demand curves for refined products are more elastic than that of crude oil.[14] The significant γ_2 coefficients for PJMW and NEPOOL certainly reflect the non-storable nature of electricity. As it is impossible to supply electricity from storage, a positive demand shock in the electricity market would require additional sources of generation which usually have a higher marginal cost. As a result, the price could shoot disproportionally higher in case of a positive demand shock than it would fall in case of a negative demand shock. It is worth noting that the estimated γ_2 coefficients for PJMW and NEPOOL are not only statistically significant

[12] In the case of crude oil, although this study uses the benchmark WTI prices which is only traded in the US, for the majority of the time during the sample period, the crude oil market is believed to be part of the broader international market (see Yergin (1991), Ewing et al (2002) and Ghoshray and Trifonova, 2014 among others).

[13] For example, reformulated regular gasoline, the price of which used in this study, is required only in cities with high smog levels and is optional elsewhere.

[14] While there is little empirical research on the supply elasticities of crude oil and refined oil products, there are some evidence that the crude oil demand elasticity is higher (i.e., the demand curve is flatter). For example, Kilian and Murphy (2014) find that the short-run price elasticity of demand for crude oil is -0.26 in the global market. Since the US oil consumption averaged about ¼ of the world consumption during the sample period, it is reasonable to assume the demand elasticity for US is close to unit (i.e. $\psi \rightarrow 1$). In comparison, Hughes, Knittel and Sperling (2008) found the short-run price elasticity for gasoline in the US for 2001-2006 is -0.04. We are cautious in comparing elasticity estimates across studies, nonetheless, they provide some indications.

at conventional levels, but also have a large reconomic effect – their magnitudes are several times higher than the other estimated coefficients, indicating a very strong asymmetric volatility effect.

The results for industrial and precious metals are also broadly consistent with findings from earlier studies in the literature and our model predictions. For example, Lucey and Tully (2006) found a statistically significant inverse leverage effect for gold and silver. Similarly, Hammoudeh and Yuan (2008) found an inverse leverage effect for gold and silver and leverage effect for copper in an EGARCH framework.[15] They attribute the difference to the fact that copper is widely used in industries and has broader sectoral linkages that are particularly prone to news impacting the world economy while gold and silver should be particularly impacted by news relating to the jewellery industry. Among industrial metals, aluminium, lead and zinc are found to have a statistically significant "inverse leverage effect". The results are largely consistent with the empirical findings on demand elasticity of industrial metals. In a recent study, Stuermer (2017) reported the following estimates for the long-run price elasticities of demand for the post World War II period. Notably, copper has the highest demand elasticity (in absolute terms) while aluminium and lead have the lowest, which agrees with our model prediction that when the demand curve is more elastic (i.e., when ψ is small), the asymmetric effect will be less pronounced.

Aluminium	Copper	Lead	Tin	Zinc
-0.160 (0.106)	-0.399*** (0.090)	-0.142*** (0.033)	-0.213*** (0.019)	-0.278*** (0.090)

Note: Reproduced from Table 6 of Stuermer (2017). Standard errors are in parenthesis. *** indicates significance at 1% level.

[15] Hammoudeh and Yuan (2008) did not acknowledge the inverse leverage effect for gold and silver, although their estimates of the EGARCH parameters (γ_1 and γ_2 in their paper) are positive and statistically significant.

There are several other interesting results in Table 2. First, while none of the estimated coefficients for the GARCH-in-mean parameter (α_1) is significantly positive at conventional levels, it is negative and statistically significant at the five percent level for wheat, natural gas, NEPOOL, PJMW, and tin. The result is in contrast with the positive GARCH-in-mean effect typically found in stock prices. A positive GARCH-in-mean is usually interpreted as the market demanding a risk premium when the expected volatility is higher. However, the negative GARCH-in-mean won't be consistent with the risk premium argument, but could be an artefact of the positive association between price shocks and volatility. A positive price shock on day *t-1*, e_{t-1}, while raising the price on day *t-1*, is associated with a higher expected volatility. As the shock dissipates on day *t*, the price goes down and consequently the return falls, resulting in a negative GARCH-in-mean. Second, the estimated coefficient for the S&P500 index (α_2) is positive and statistically significant, mostly at the one percent level, for all agricultural products, industrial metals, all energy products except electricity, and one of the precious metals – palladium. The result is consistent with the argument that investing in stock markets represents an opportunity cost of investing in commodity markets. It could also be driven by the same underlying demand force in that when the demand for commodities increases, the expected return in stock market is likely to rise. Notably, gold is the only commodity that exhibits a negative return relationship with the S&P500 index, probably reflecting gold as the "safe heaven" of investment and used for hedging. Third, as one would expect, the estimated coefficient for US dollar index (α_3) is uniformly negative and significant at the one percent level for all commodities but electricity and gasoline, indicating the importance of the dollar exchange rate in commodity price fluctuations. Finally, the estimated coefficient for the GARCH term in the variance equation (γ_3) is either higher than or close to 0.9 for the vast majority of commodities, indicating a high degree of volatility persistence in the data.

The estimation results for TGARCH with jumps models are reported in Table 3. The results are similar to those reported in Table 2. The estimated γ_2 coefficient for natural gas becomes significantly negative at the five percent level, indicating an "inverse leverage effect" when jumps are accounted for. The estimates for other commodities are largely consistent with those reported in Table 2. As for the parameters relating to the jump process, both λ and δ are statistically significant for all commodities. However, θ is statistically significant only for energy products, corn, soybean, and gold. Recall that θ is the mean for the distribution of jump size. The insignificant θ parameter for industrial metals, silver, palladium, platinum, and wheat perhaps explains why the results for these products are so similar in specifications with and without jumps. Although not reported, models with jumps also fit the data better as they generally have a lower AIC and SIC than their counterparts without jumps.

Table 4 reports the TGARCH estimates using 3-month futures price data for energy products and industrial metals.[16] The results with and without jumps are quite similar. Notably, the estimated γ_2 coefficients for both gasoline and heating oil turned positive and statistically significant in the three-month futures data, and the coefficient of crude oil continues to be positive. In other words, as the time-to-maturity of the future contracts lengthens, the leverage effect becomes more pronounced. The result is consistent with our second conjecture that as the contract horizon lengthens, both the demand and supply relationship become more elastic and the return volatility is negatively associated with shocks as illustrated in Figure 1d. For industrial metals, in the model without jumps, the estimated γ_2 becomes statistically insignificant for lead in

[16] We don't have futures data for agricultural products and precious metals.

the three-month futures data while it continues to be significant for aluminium and zinc. In the model with jumps, the estimated γ_2 is statistically significant only for aluminium. Taken together, the results in Table 4 supports our second conjecture that the inverse-leverage effect is weaker in futures market than it is in spot market.

To give a clearer picture of the magnitude of asymmetries in volatility, in Table 5 we report the implied log percentages in conditional volatility, $\ln(\sigma_{t+1}/\sigma_t)$, following various return shocks, e_t, for the 12 commodities that were found to have a statistically significant (at 5 percent level) asymmetric volatility effect in the TGARCH model of Table 2. The conditional volatility on day t, σ_t, is assumed at their respective unconditional levels. Figure 2 plots these log price changes which are commonly referred to as news impact curves. The differences in the magnitudes of the impact curves partly reflect the differences in the estimated GARCH parameters, but more importantly are determined by the unconditional volatilities since, for example, a 5% shock corresponds to two standard deviations for gasoline but more than five standard deviations for gold. For this reason, we plot the electricity curves separately to make a visible comparison because the electricity price is considerably more volatile than other commodities. Clearly, with the exception of crude oil, the news impact curves for all other commodities in the spot markets, shown in the first panel of Figure 2, are *J*-shaped as opposed to the reverse *J*-shape reported by Ederington and Guan (2010) for stock market. Both large positive and negative shocks increase the conditional volatility on next day, but the impact of positive shocks is stronger. For instance, the conditional volatility for heating oil increases by 17% following a positive price shock of 5% whereas it increases by 10% following a 5% price drop. Perhaps not surprisingly, the conditional volatility decreases when the market is tranquil and achieves greatest reduction when the return surprise (e_t) is zero.

In spot markets only crude oil displays a reverse-*J* shaped news impact curve similar to those typically observed in stock markets (as shown in the last panel of Figure 2). According to the estimates, the conditional volatility for crude oil would increase 12% following a negative price shock of 5% compared with an increase of 7% following a 5% price spike. In next section, we provide a discussion of the crude oil result.

Lastly, since some of the recent literature (see Bauer and Dimpfl, 2018 for example) reports that the positive return-volatility relationship in commodity markets has weakened since 2005 possibly due to the financialization of commodities as argued by Tang and Xiong (2010), we investigate whether it holds in our data as well. Following Bauer and Dimpfl (2018), we conduct the analysis by breaking down the full sample into two periods – before and after January 1, 2005. The estimated γ_2 coefficients are reported in Table 6. For ease of comparison, we reproduce the results of the full sample in the last column. Indeed, the estimated γ_2 coefficient is negative and statistically significant in 12 out of 19 commodities before 2005 while remains so in only six of the commodities. It was insignificant for crude oil before 2005 and turns to positive and highly significant after 2005. Perhaps influenced by crude oil, the coefficient for gasoline changed from negative to positive, and was statistically significant at the one percent level in both periods. For heating oil, natural gas, lead, tin, zinc, and palladium, it was negative and statistically significant at the five percent level before 2005 but turns insignificant after 2005. Thus, our results also provide support to the idea that the financialization of commodities (particularly energy and industrial metals) may have played a role in the weakening of the positive return-volatility association and strengthening of the negative association similar to that in the equity markets.

4.3 Why is Crude Oil Different?

The positive association of return shocks and volatility for crude oil is somewhat puzzling. Nonetheless, it could also be consistent with our model predictions considering the unique characteristics of the crude oil market. First, as argued earlier, the demand curve for crude oil in the context of US market is flatter (i.e. ψ is smaller) than refined products such as gasoline or heating oil, hence the asymmetric relationship is less pronounced even if the supply relationship is convex (Figure 1C).

Second, the market structure of crude oil is best described by a dominant firm model with a competitive fringe where OPEC (or Saudi Arabia) has the market power to influence the price by adjusting production[17]. As widely acknowledged, the industry cost structure is characterized by high capital cost and low variable cost. When there are spare production capacities, the marginal cost of crude oil production in many of the major oil producing countries, such as Saudi Arabia, is fairly low. [18] According to the US Energy Information Administration, the direct lifting cost (the operating cost) of producing one barrel of crude oil was \$8-10 in the US and \$4-5 in the Middle East during 2007-2009, which was far below the prevailing oil prices at same period. [19] When there is a positive demand shock, OPEC has the capacity to increase production from its low cost fields to "stabilize" the price in order to maximize its long-term profit. So the price on the margin may not change much. Whereas in times of a negative demand shock (i.e. the price is relatively low), the possibility of OPEC reducing production or the high cost fringe firms shut-in production in response to the lower

[17] See, for example, Alhajji and Huettner (2000) and Almoguera et al (2011). In this case, OPEC or Saudi Arabia can be considered the dominant firm and other producers the competitive fringe.
[18] For most of time during the sample period, there are significant spare production capacities held by OPEC countries, particularly by Saudi Arabia (See EIA short-term outlook data).
[19] EIA, "Performance profiles of major energy producers" financial tables. Also, see Anderson et al (2014) "Hotelling under pressure".

price could lead to large price increases. Corroborating this conjecture, a recent paper by Plante (2018) provides empirical evidence of the connection between OPEC and oil price volatility. He documents a strong correlation between the number of news articles surrounding OPEC events and measures of oil price volatility. In sum, because of OPEC's ability to withhold the low cost capacities from the market, the perceived supply relationship might be concave for crude oil in the sense that when the demand is relatively low a small quantity change might result in a larger price change whereas when the demand is high the same quantity change only leads to a smaller price change. As a counterfactual exercise, we estimated the TGARCH model for the period of 6/1/2008—9/30/2008 during which the spare capacities were nearly running out, and the estimated γ_2 coefficients were negative although not statistically significant for both the front-month and 3-month futures.

Another seemingly plausible explanation is the spill-over effect from financial markets. Tang and Xiong (2010) argue that the daily price of crude oil is increasingly influenced by index investment and consequently the volatility has a stronger correlation with that of the stock market. However, our test for volatility spillover from the stock market to the crude oil market, using Hong's (2001) methodology does not find strong evidence of volatility spillover from the stock market to crude oil during the sample period.

In summary, the GARCH estimation of daily prices find statistically significant evidence of asymmetric volatility in more than half the commodity spot prices. The result is robust whether jumps are accounted for or not. Contrary to the "leverage effect" found in the stock market, the asymmetry in commodity markets shows an "inverse leverage effect" in which positive shocks have stronger impact on the conditional volatility than negative shocks of the same magnitude. Consistent with the theoretical predictions in section 2, the “inverse leverage effect” is stronger for spot prices than for 3-month forward prices and for those with a geographically constrained supply or more costly storage.

5. Historical Volatility

While the time-series models provide evidence of asymmetric relationship between the forecast daily volatility and return shocks, we are also interested in how the e*x post* historical volatility behaves in the presence of positive and negative return shocks. To this end, we calculate the historical return volatility using monthly standard deviations of the returns of spot prices and estimate the following model for each commodity:

$$\omega_m = \lambda_0 + \lambda_1 r_m + \lambda_2 \omega_{m-1} + \eta_t \ , \qquad (12)$$

where ω_m is the standard deviation of the daily returns in month m and r_m is the log monthly return calculated on the monthly average prices. We include the lagged variable ω_{m-1} so that λ_1 measures the effect of the mean monthly returns on historical volatility after controlling last month’s volatility. A positive λ_1 indicates a price increase (decrease) is associated with a higher (lower) volatility, and therefore providing evidence of the “inverse leverage effect”.

The estimation results using spot prices are reported in Table 6. The *p*-values are based on the heteroscedasticity and autocorrelation consistent Newey-West standard errors. Overall, the evidence of “inverse leverage effect” is much weaker as most of the estimated coefficients of λ_1 is not statistically significant. Nonetheless, it is positive and significant at least in the 10 percent level for wheat, natural gas, electricity (NEPOOL and PJMW) and gold. Thus, for these commodities, the inverse leverage effect found in the GARCH models is also evident in the monthly historical volatility.

The estimated λ_1 for crude oil is negative and statistically significant at the five percent level. The volatility for crude oil tends to decrease (increase) when the price goes up (down), which is consistent with the asymmetric time-series models.

6. Conclusion

This paper examines how return volatility responds differently following positive and negative shocks in a range of commodities including agricultural products, energy, industrial metals and precious metals. There are two major contributions contained in this paper. First, we develop a commodity price volatility model in which the equilibrium price is determined by a stochastic demand and an upward sloping cost curve. We show that the volatility of price changes can be positively or negatively related to demand shocks depending on the demand and supply elasticities. This model of volatility is more general compared to earlier models of commodity volatility which are often limited to a particular sector.

Second, we empirically document the pattern of asymmetric volatility using both time-series techniques and monthly historical volatility measures. Employing asymmetric GARCH models, we find a statistically significant "inverse leverage effect" – the volatility is higher following a positive return shock – in more than half of the spot price series. The results hold when jumps were accounted for. Only crude oil is found to have a "leverage effect" – the volatility is higher following a negative return shock. The inverse leverage effect is weaker in the price of 3-month futures and historical volatility measures based on monthly standard deviations of daily returns. Both results are consistent with our theoretical predictions.

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Table 1 Descriptive Statistics of Daily Commodity Returns

	Commodity	Mean (%)	Std. deviation (%)	Maximum (%)	Minimum (%)	Skewness	Kurtosis	Sample Period	No. Obs
Agriculture	Corn	0.0031	1.6815	10.9071	-12.306	-0.2792	7.4317	1/2/1980-2/28/2018	9730
	Soybean	0.0051	1.5082	7.5730	-16.7413	-0.6477	8.9960	1/2/1980-2/28/2018	9730
	Wheat	0.0033	1.6693	28.0302	-26.1953	0.1128	22.7020	1/2/1980-2/28/2028	9730
Energy Products	*Front month*								
	Crude oil	0.0084	2.3756	16.4097	-40.0478	-0.7156	17.4119	4/4/1983-2/28/2018	8755
	Gasoline	0.0115	2.4843	21.6646	-30.9823	-0.4625	12.6134	1/2/1985-2/28/2018	8315
	Heating oil	0.0089	2.2507	13.965	-39.0297	-1.3777	22.0822	1/2/1980-2/28/2018	9569
	Natural gas	0.0002	3.5392	32.4354	-37.5749	0.2262	9.9792	2/1/1994-2/28/2018	6032
	3-month contract								
	Crude oil	0.0085	1.9826	12.115	-32.8206	-0.6481	14.7192	4/4/1983-2/28/2018	8755
	Gasoline	0.0122	2.0197	16.3135	-26.0803	-0.2394	10.3273	1/2/1985-2/28/2018	8315
	Heating oil	0.0080	1.8672	9.6584	-30.8516	-0.6205	13.2420	1/2/1980-2/28/2018	9569
	Natural gas	0.0032	2.7688	21.5196	-31.1209	-0.1227	10.4230	2/1/1994-2/28/2018	6032
	Electricity day-ahead								
	NEPOOL day-ahead	-0.0217	17.9123	126.6667	-109.5600	0.3385	9.0938	1/1/2004-2/28/2018	3406
	PJMW day-ahead	-0.0199	17.7414	107.7794	-153.0240	-0.2553	11.8866	1/2/2001-2/28/2018	4366
Industrial Metals	*Cash*								
	Aluminum	0.0092	1.3013	6.0679	-8.2551	-0.2104	5.3665	8/3/1993-2/28/2018	6207
	Copper	0.0202	1.6324	11.7259	-10.4755	-0.1888	7.8252	8/3/1993-2/28/2018	6207
	Lead	0.0298	1.9390	13.0072	-13.1992	-0.1562	6.6057	8/3/1993-2/28/2018	6207
	Nickel	0.0162	2.2035	13.3096	-18.3586	-0.1360	6.7808	8/3/1993-2/28/2018	6207

	Tin	0.0241	1.5786	15.3854	-11.4532	-0.1981	10.0674	8/3/1993-2/28/2018	6207
	Zinc	0.0214	1.7502	9.9490	-12.6185	-0.2718	6.9038	8/3/1993-2/28/2018	6207
Industrial Metals	*3-month contract*								
	Aluminum	0.0088	1.2347	5.9131	-8.2472	-0.2286	5.5708	8/3/1993-2/28/2018	6216
	Copper	0.0203	1.5523	11.8805	-10.4003	-0.1787	7.8741	8/3/1993-2/28/2018	6216
	Lead	0.0293	1.8174	12.6752	-12.8495	-0.2107	7.1980	8/3/1993-2/28/2018	6216
	Nickel	0.0164	2.1405	13.0603	-18.1061	-0.1567	6.8363	8/3/1993-2/28/2018	6216
	Tin	0.0238	1.5360	14.2533	-11.4346	-0.1852	10.2592	8/3/1993-2/28/2018	6216
	zinc	0.021	1.6794	9.6564	-11.0098	-0.2247	6.6909	8/3/1993-2/28/2018	6216
	Gold	0.0164	0.9904	7.3820	-10.1624	-0.3799	10.7741	1/3/1990-2/28/2018	7186
Precious Metals	Palladium	0.0288	1.9966	15.8406	-17.8590	-0.1667	9.6283	1/3/1990-2/28/2018	7186
	Platinum	0.0093	1.3576	13.0620	-16.7723	-0.3969	12.2926	1/3/1990-2/28/2018	7186
	Silver	0.0163	1.8672	18.2786	-18.6926	-0.4067	12.9917	1/3/1990-2/28/2018	7186
Stocks	S&P 500 index	0.0333	1.1004	10.9572	-22.8997	-1.1709	30.2671	1/2/1980-2/28/2018	9730

Table 2 TGARCH estimates of spot price returns

	α_1	α_2	α_3	γ_1	γ_2	γ_3
Agricultural products						
Corn	-0.0404	0.0828	-0.2004	0.0832	-0.0005	0.9108
	(0.1795)	(0.0000)	(0.0000)	(0.0000)	(0.9594)	(0.0000)
Soybean	-0.0169	0.0772	-0.2999	0.0842	**-0.0467**	0.9339
	(0.6067)	(0.0000)	(0.0000)	(0.0000)	**(0.0000)**	(0.0000)
Wheat	-0.0530	0.0566	-0.1438	0.0944	**-0.0211**	0.9140
	(0.0506)	(0.0000)	(0.0000)	(0.0000)	**(0.0427)**	(0.0000)
Energy products						
Crude oil	0.0435	0.1578	-0.3711	0.0494	**0.0258**	0.9402
	(0.0539)	(0.0000)	(0.0000)	(0.0000)	**(0.0000)**	(0.0000)
Gasoline	0.0368	0.1803	-0.4073	0.0503	0.0005	0.9407
	(0.3319)	(0.0000)	(0.0000)	(0.0000)	(0.9503)	(0.0000)
Heating oil	0.0267	0.1401	-0.2770	0.0854	**-0.0273**	0.9316
	(0.2608)	(0.0000)	(0.0000)	(0.000)	**(0.0008)**	(0.0000)
Natural gas	-0.1311	0.0716	-0.2767	0.0834	-0.0209	0.9055
	(0.0081)	(0.0245)	(0.0005)	(0.0000)	(0.0685)	(0.0000)
PJMW	-0.3501	-0.0231	-0.5124	0.3025	**-0.2514**	0.7406
	(0.0000)	(0.8769)	(0.1485)	(0.0000)	**(0.0000)**	(0.0000)
NEPOOL	-0.2033	-0.0778	-0.5224	0.4120	**-0.2966**	0.7568
	(0.0000)	(0.4534)	(0.0572)	(0.0000)	**(0.0000)**	(0.0000)
Industrial Metals						
Aluminium	0.0290	0.1738	-0.6067	0.0572	**-0.0230**	0.9385
	(0.6427)	(0.0000)	(0.0000)	(0.0000)	**(0.0074)**	(0.0000)
Copper	0.0707	0.2396	-0.7019	0.0524	0.0013	0.9360
	(0.1360)	(0.0000)	(0.0000)	(0.0000)	(0.8791)	(0.0000)
Lead	0.0491	0.1676	-0.6456	0.0488	**-0.0172**	0.9564
	(0.2494)	(0.0000)	(0.0000)	(0.0000)	**(0.0217)**	(0.0000)
Nickel	0.0342	0.2589	-0.7808	0.0467	-0.0057	0.9433
	(0.5407)	(0.0000)	(0.0000)	(0.0000)	(0.4649)	(0.0000)
Tin	0.0876	0.0944	-0.4018	0.0871	-0.0047	0.9138
	(0.0078)	(0.0000)	(0.0000)	(0.0000)	(0.6970)	(0.0000)
Zinc	0.0012	0.1716	-0.5474	0.0418	**-0.0161**	0.9651
	(0.9736)	(0.0000)	(0.0000)	(0.0000)	**(0.0091)**	(0.0000)
Precious Metals						
Gold	0.0509	-0.0363	-0.6178	0.0798	**-0.0488**	0.9454
	(0.0918)	(0.0000)	(0.0000)	(0.0000)	**(0.0000)**	(0.0000)
Silver	0.0662	-0.0027	-0.5449	0.0634	**-0.0425**	0.9577
	(0.0341)	(0.8462)	(0.0000)	(0.0000)	**(0.0000)**	(0.0000)
Platinum	0.0446	0.0209	-0.5603	0.0714	-0.0161	0.9334
	(0.2239)	(0.0702)	(0.0000)	(0.0000)	(0.0730)	(0.0000)
Palladium	0.0285	0.0735	-0.5084	0.1451	**-0.0385**	0.8771
	(0.3159)	(0.0000)	(0.0000)	(0.0000)	**(0.0095)**	(0.0000)

Note: The table reports the estimation results of log returns calculated from spot prices. The numbers in parenthesis are *P*-values based on Student's *t* distribution.

Table 3 Spot Returns TGARCH with Jumps

	α_2	α_3	γ_1	γ_2	γ_3	δ	θ	λ
Agriculture								
Corn	0.0862	-0.2038	0.0697	-0.0049	0.9060	2.5868	-0.4654	0.0831
	(0.0000)	(0.0000)	(0.0000)	(0.5310)	(0.0000)	(0.0000)	(0.0015)	(0.0000)
Soybean	0.0778	-0.3064	0.0657	**-0.0348**	0.9322	2.1262	-0.3949	0.1019
	(0.0000)	(0.0000)	(0.0000)	**(0.0000)**	(0.0000)	(0.0000)	(0.0002)	(0.0000)
Wheat	0.0619	-0.1477	0.0721	**-0.0189**	0.8902	2.2426	0.1231	0.1814
	(0.0000)	(0.0000)	(0.0000)	**(0.0098)**	(0.0000)	(0.0000)	(0.0929)	(0.0000)
Energy								
Crude oil	0.1502	-0.3718	0.0451	**0.0231**	0.9325	3.6098	-0.6790	0.0611
	(0.0000)	(0.0000)	(0.0000)	**(0.0012)**	(0.0000)	(0.0000)	(0.0047)	(0.0000)
Gasoline	0.1625	-0.4053	0.0278	0.0062	0.9508	4.7806	-0.8502	0.0690
	(0.0000)	(0.0000)	(0.0000)	(0.2239)	(0.0000)	(0.0000)	(0.0020)	(0.0000)
Heating oil	0.1459	-0.2819	0.0633	**-0.0208**	0.9319	3.6059	-0.5946	0.0754
	(0.0000)	(0.0000)	(0.0000)	**(0.0015)**	(0.0000)	(0.0000)	(0.0044)	(0.0000)
Natural gas	0.0732	-0.2817	0.0704	**-0.0252**	0.9079	6.6183	2.2000	0.0481
	(0.0262)	(0.0007)	(0.0000)	**(0.0095)**	(0.0000)	(0.0000)	(0.0003)	(0.0001)
PJMW	-0.0282	-0.6729	0.2139	**-0.1975**	0.7960	16.7878	13.9674	0.1208
	(0.8408)	(0.0608)	(0.0000)	**(0.0000)**	(0.0000)	(0.0000)	(0.0000)	(0.0000)
NE Pool	-0.0420	-0.6838	0.2461	**-0.1695**	0.7575	19.964	17.3105	0.0947
	(0.6458)	(0.006)	(0.0000)	**(0.0000)**	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Industrial Metals								
Aluminium	0.1707	-0.6054	0.0509	**-0.0206**	0.9358	1.5189	0.0464	0.1392
	(0.0000)	(0.0000)	(0.0000)	**(0.0051)**	(0.0000)	(0.0000)	(0.6551)	(0.0056)
Copper	0.2442	-0.7054	0.0498	0.0000	0.9268	1.8340	-0.1860	0.1476
	(0.0000)	(0.0000)	(0.0000)	(0.9985)	(0.0000)	(0.0000)	(0.1056)	(0.0001)
Lead	0.1635	-0.6532	0.0455	**-0.0132**	0.9481	2.3142	0.1220	0.1213
	(0.0000)	(0.0000)	(0.0000)	**(0.0441)**	(0.0000)	(0.0000)	(0.3815)	(0.0001)
Nickel	0.2547	-0.8000	0.0421	-0.0063	0.9373	2.7329	0.0600	0.1534
	(0.0000)	(0.0000)	(0.0000)	(0.3460)	(0.0000)	(0.0000)	(0.6847)	(0.0008)
Tin	0.0960	-0.4163	0.0684	0.0054	0.8752	2.1688	-0.1387	0.1612
	(0.0000)	(0.0000)	(0.0000)	(0.5716)	(0.0000)	(0.0000)	(0.1506)	(0.0000)
Zinc	0.1697	-0.5422	0.0307	-0.0090	0.9673	2.3330	-0.2184	0.0810
	(0.0000)	(0.0000)	(0.0000)	(0.0785)	(0.0000)	(0.0000)	(0.2361)	(0.0001)
Precious Metals								
Gold	-0.0385	-0.6408	0.0598	**-0.0392**	0.9375	1.4268	-0.1740	0.1148
	(0.0000)	(0.0000)	(0.0000)	**(0.0000)**	(0.0000)	(0.0000)	(0.0186)	(0.0000)
Silver	0.0077	-0.6207	0.0545	**-0.0323**	0.9424	2.6887	-0.0549	0.1144
	(0.5932)	(0.0000)	(0.0000)	**(0.0000)**	(0.0000)	(0.0000)	(0.6878)	(0.0000)
Palladium	0.0757	-0.5096	0.0983	**-0.0316**	0.8657	2.7416	-0.1275	0.1563
	(0.0000)	(0.0000)	(0.0000)	**(0.0001)**	(0.0000)	(0.0000)	(0.2012)	(0.0000)
Platinum	0.0231	-0.5642	0.0608	**-0.0179**	0.9272	1.6762	-0.0720	0.1434
	(0.059)	(0.0000)	(0.0000)	**(0.0085)**	(0.0000)	(0.0000)	(0.4271)	(0.0000)

p-values based on Student's t distribution are in parenthesis. λ, θ and δ are respectively the jump intensity, the mean and variance of jump size parameters.

Table 4 TGARCH estimates of 3-month futures

	α_2	α_3	γ_1	γ_2	γ_3
Energy (without jumps)					
Crude oil	0.1385 (0.0000)	-0.3263 (0.0000)	0.0423 (0.0000)	**0.0358** **(0.0000)**	0.9402 (0.0000)
Gasoline	0.1462 (0.0000)	-0.3303 (0.0000)	0.0293 (0.0000)	**0.0209** **(0.0006)**	0.9554 (0.0000)
Heating oil	0.1351 (0.0000)	-0.3043 (0.0000)	0.0457 (0.0000)	**0.0183** **(0.0066)**	0.9434 (0.0000)
Natural gas	0.0525 (0.0458)	-0.2064 (0.0018)	0.0510 (0.0000)	0.0049 (0.6026)	0.9319 (0.0000)
Energy (with jumps)					
Crude oil	0.1364 (0.0000)	-0.3237 (0.0000)	0.0393 (0.0000)	**0.0320** **(0.0000)**	0.9314 (0.0000)
Gasoline	0.1425 (0.0000)	-0.3357 (0.0000)	0.0249 (0.0000)	**0.0149** **(0.0019)**	0.9527 (0.0000)
Heating oil	0.1340 (0.0000)	-0.2989 (0.0000)	0.0386 (0.0000)	**0.0129** **(0.0102)**	0.9449 (0.0000)
Natural gas	0.0569 (0.035)	-0.2323 (0.0011)	0.0397 (0.0000)	0.0052 (0.4994)	0.9311 (0.0000)
Industrial Metals (without jumps)					
Aluminium	0.1622 (0.0000)	-0.5746 (0.0000)	0.0505 (0.0000)	**-0.0171** **(0.0313)**	0.9455 (0.0000)
Copper	0.2249 (0.0000)	-0.6516 (0.0000)	0.0449 (0.0000)	0.0067 (0.3696)	0.9437 (0.0000)
Lead	0.1445 (0.0000)	-0.5771 (0.0000)	0.0392 (0.0000)	-0.0053 (0.4419)	0.9613 (0.0000)
Nickel	0.2520 (0.0000)	-0.7677 (0.0000)	0.0443 (0.0000)	-0.0030 (0.7092)	0.9447 (0.0000)
Tin	0.0721 (0.0000)	-0.3589 (0.0000)	0.0851 (0.0000)	0.0016 (0.8917)	0.9153 (0.0000)
Zinc	0.1602 (0.0000)	-0.5053 (0.0000)	0.0391 (0.0000)	**-0.0131** **(0.0288)**	0.9672 (0.0000)
Industrial Metals (with jumps)					
Aluminium	0.1597 (0.0000)	-0.5732 (0.0000)	0.0446 (0.0000)	**-0.0150** **(0.0499)**	0.9439 (0.0000)
Copper	0.2315 (0.0000)	-0.6550 (0.0000)	0.0425 (0.0000)	0.0041 (0.5628)	0.9366 (0.0000)
Lead	0.1392 (0.0000)	-0.5754 (0.0000)	0.0362 (0.0000)	-0.0024 (0.7089)	0.9551 (0.0000)
Nickel	0.2444 (0.0000)	-0.778 (0.0000)	0.0396 (0.0000)	-0.0050 (0.483)	0.9410 (0.0000)
Tin	0.0722 (0.0000)	-0.3745 (0.0000)	0.0674 (0.0000)	0.0091 (0.2502)	0.8787 (0.0000)
Zinc	0.1559 (0.0000)	-0.5060 (0.0000)	0.0301 (0.0000)	-0.0062 (0.2060)	0.9669 (0.0000)

Note: The numbers in parenthesis are *P*-values based on Student's t distribution.

Table 5 **Percentage change in conditional volatility following a return shock based on TGARCH estimation**

	Surprise market return on day t, e_t												
	-6%	-5%	-4%	-3%	-2%	-1%	0%	1%	2%	3%	4%	5%	6%
Soybean	21.18	14.86	9.02	3.95	-0.01	-2.55	-3.42	-1.48	3.94	11.83	21.13	31.00	40.90
Wheat	31.07	22.62	14.45	7.03	0.96	-3.07	-4.49	-2.67	2.42	9.90	18.79	28.30	37.90
Heating oil	14.80	9.88	5.45	1.71	-1.14	-2.93	-3.54	-2.65	-0.05	4.00	9.17	15.11	21.54
Aluminium	25.54	18.37	11.64	5.69	0.96	-2.11	-3.18	-1.41	3.55	10.86	19.57	28.92	38.38
Lead	11.07	7.40	4.17	1.51	-0.48	-1.71	-2.13	-1.48	0.43	3.46	7.42	12.09	17.25
Zinc	11.84	8.06	4.74	1.99	-0.07	-1.35	-1.78	-1.08	0.97	4.21	8.42	13.36	18.80
Gold	36.77	27.63	18.67	10.39	3.50	-1.14	-2.78	1.34	12.01	25.90	40.51	54.61	67.75
Silver	7.98	5.09	2.59	0.56	-0.94	-1.86	-2.17	-1.23	1.51	5.76	11.16	17.34	23.99
Palladium	30.57	21.85	13.36	5.60	-0.78	-5.05	-6.56	-4.52	1.13	9.33	18.94	29.07	39.19
Crude oil	17.53	12.08	7.13	2.92	-0.33	-2.38	-3.09	-2.62	-1.26	0.94	3.85	7.37	11.36

	Surprise market return on day t, e_t												
	-24%	-20%	-16%	-12%	-8%	-4%	0%	4%	8%	12%	16%	20%	24%
PJMW	-9.07	-10.81	-12.29	-13.46	-14.32	-14.84	-15.02	-13.99	-11.03	-6.45	-0.67	5.89	12.89
NEPOOL	-1.83	-5.23	-8.19	-10.62	-12.43	-13.55	-13.93	-12.59	-8.78	-3.00	4.11	11.97	20.16

Notes: This table illustrates the log percentage changes ($\ln\sigma_{t+1}/\sigma_t$) following a surprise market return on day t based on the TGARCH estimation reported in Table 2. The volatility before the shock was assumed at its unconditional mean level.

Table 6 TGARCH estimates of spot price returns before and after 2005

	Pre-2005	Post-2005	Full Sample
Agriculture			
Corn	0.0018 (0.8905)	-0.0061 (0.6258)	-0.0005 (0.9594)
Soybean	**-0.0507** **(0.0000)**	**-0.0368** **(0.0019)**	**-0.0467** **(0.0000)**
Wheat	-0.0221 (0.1302)	-0.0243 (0.1703)	**-0.0211** **(0.0427)**
Energy			
Crude oil	0.0102 (0.3503)	**0.0582** **(0.0000)**	**0.0258** **(0.0000)**
Gasoline	**-0.0311** **(0.0041)**	**0.0361** **(0.0004)**	0.0005 (0.9503)
Heating oil	**-0.0521** **(0.0000)**	0.0099 (0.3685)	**-0.0273** **(0.0008)**
Natural Gas	**-0.0395** **(0.0461)**	0.0012 (0.9280)	-0.0209 (0.0685)
PJMW	**-0.0830** **(0.0459)**	**-0.3147** **(0.0000)**	**-0.2514** **(0.0000)**
NEPool	**-0.4400** **(0.0057)**	**-0.2798** **(0.0000)**	**-0.2966** **(0.0000)**
Industrial Metals			
Aluminium	-0.0206 (0.1414)	**-0.0306** **(0.0394)**	**-0.0230** **(0.0074)**
Copper	-0.0217 (0.1244)	0.0192 (0.1458)	0.0013 (0.8791)
Lead	**-0.0399** **(0.0088)**	-0.0097 (0.4094)	**-0.0172** **(0.0217)**
Nickel	0.0026 (0.8166)	-0.0180 (0.1473)	-0.0057 (0.4649)
Tin	**-0.0447** **(0.0288)**	0.0271 (0.1150)	-0.0047 (0.6970)
Zinc	**-0.0378** **(0.0012)**	-0.0005 (0.9609)	**-0.0161** **(0.0091)**
Precious Metals			
Gold	**-0.0593** **(0.0001)**	**-0.0460** **(0.0000)**	**-0.0488** **(0.0000)**
Silver	**-0.0607** **(0.0000)**	**-0.0329** **(0.0013)**	**-0.0425** **(0.0000)**
Platinum	-0.0128 (0.2872)	-0.0171 (0.1978)	-0.0161 (0.0730)
Palladium	**-0.1168** **(0.0019)**	-0.0076 (0.5831)	**-0.0385** **(0.0095)**

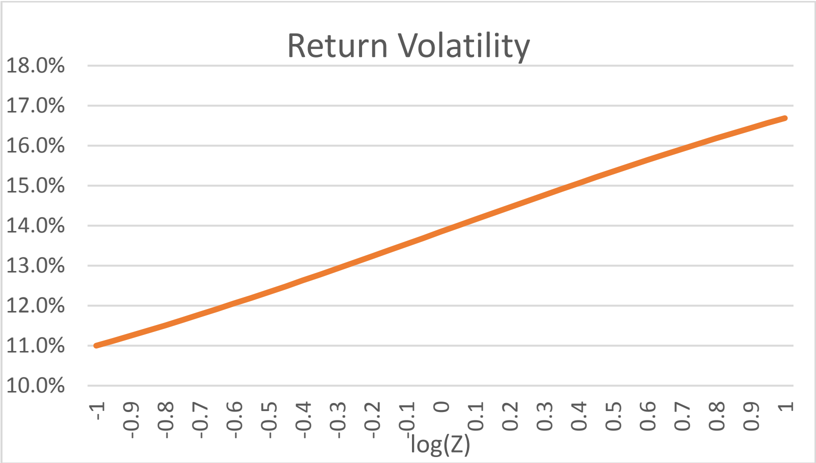
Note: The table reports the estimation results of log returns calculated from spot prices before and after January 1, 2005. The numbers in parenthesis are *P*-values based on Student's *t* distribution. The last column is reproduced from column 5 (γ_2) of Table 2.

Table 7 Estimates of Monthly Historical Volatility

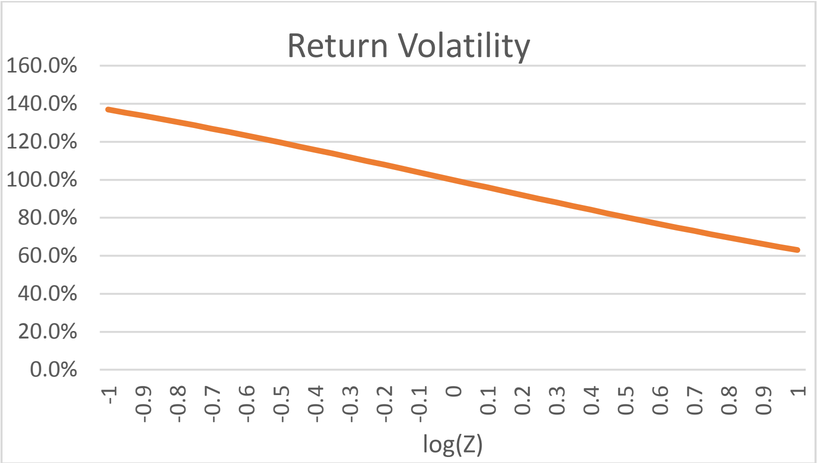
	λ_1	λ_2	Adj. R_sq	N
		Agricultural products		
Corn	0.0004 (0.9521)	0.6164 (0.0000)	0.376	457
Soybean	0.0052 (0.5073)	0.6087 (0.0000)	0.358	457
Wheat	**0.0222** **(0.0045)**	0.449- (0.0000)	0.214	457
		Energy products		
Crude oil	**-0.0295** **(0.0005)**	0.6455 (0.0000)	0.476	418
Gasoline	-0.0103 (0.1318)	0.5313 (0.0000)	0.293	397
Heating oil	-0.0117 (0.1953)	0.5955 (0.0000)	0.373	457
Natural gas	0.0172 (0.0657)	0.5512 (0.0000)	0.308	288
NEPOOL	**0.1443** **(0.0000)**	0.5737 (0.0000)	0.391	205
PJMW	**0.2578** **(0.0000)**	0.6631 (0.0000)	0.533	205
		Industrial metals		
Aluminium	0.0020 (0.8138)	0.5599 (0.0000)	0.309	294
Copper	-0.0157 (0.2550)	0.6107 (0.0000)	0.407	294
Lead	0.0004 (0.9627)	0.6820 (0.0000)	0.463	294
Nickel	-0.0053 (0.5895)	0.5220 (0.0000)	0.276	294
Tin	-0.0081 (0.1744)	0.5948 (0.0000)	0.361	294
Zinc	-0.0037 (0.7100)	0.7040 (0.0000)	0.498	294
		Precious metals		
Gold	**0.0166** **(0.0246)**	0.6614 (0.0000)	0.446	337
Silver	0.0117 (0.2921)	0.6297 (0.0000)	0.389	337
Platinum	0.0000 (0.9981)	0.5286 (0.0000)	0.275	337
Palladium	0.0146 (0.1786)	0.5714 (0.0000)	0.324	337

Note: The table reports estimation results of equation (12) using spot prices. *p*-values based on Newey-West robust standard errors are in parenthesis.

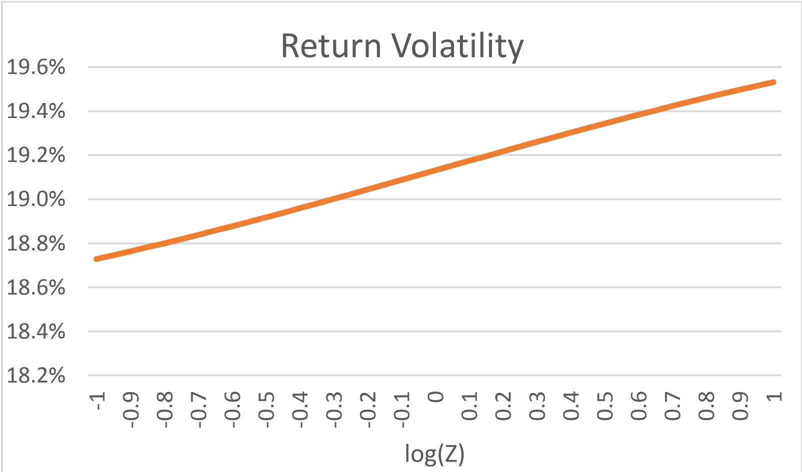
Figure 1 Relationship between volatility and demand shocks



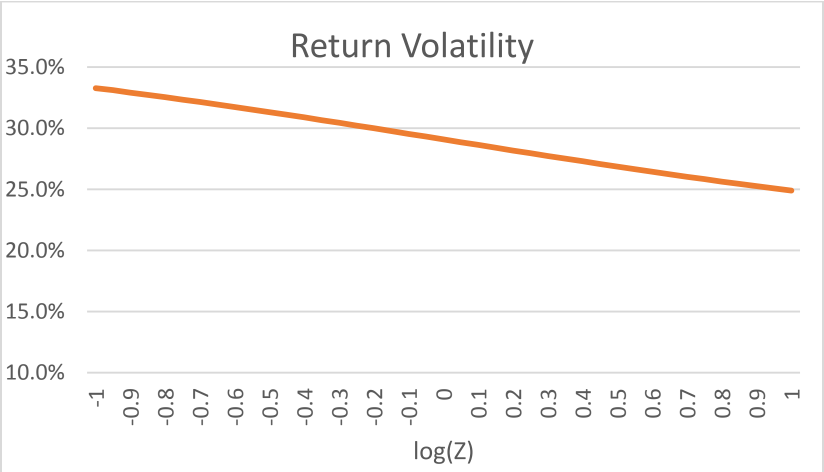
(Figure 1a: $\psi = 5, \gamma = 1.5, \sigma = 20\%, C_0 = 1$, and $\epsilon = 1$)



(Figure 1b: $\psi = 5, \gamma = 0.1, \sigma = 20\%, C_0 = 1$, and $\epsilon = 1$)

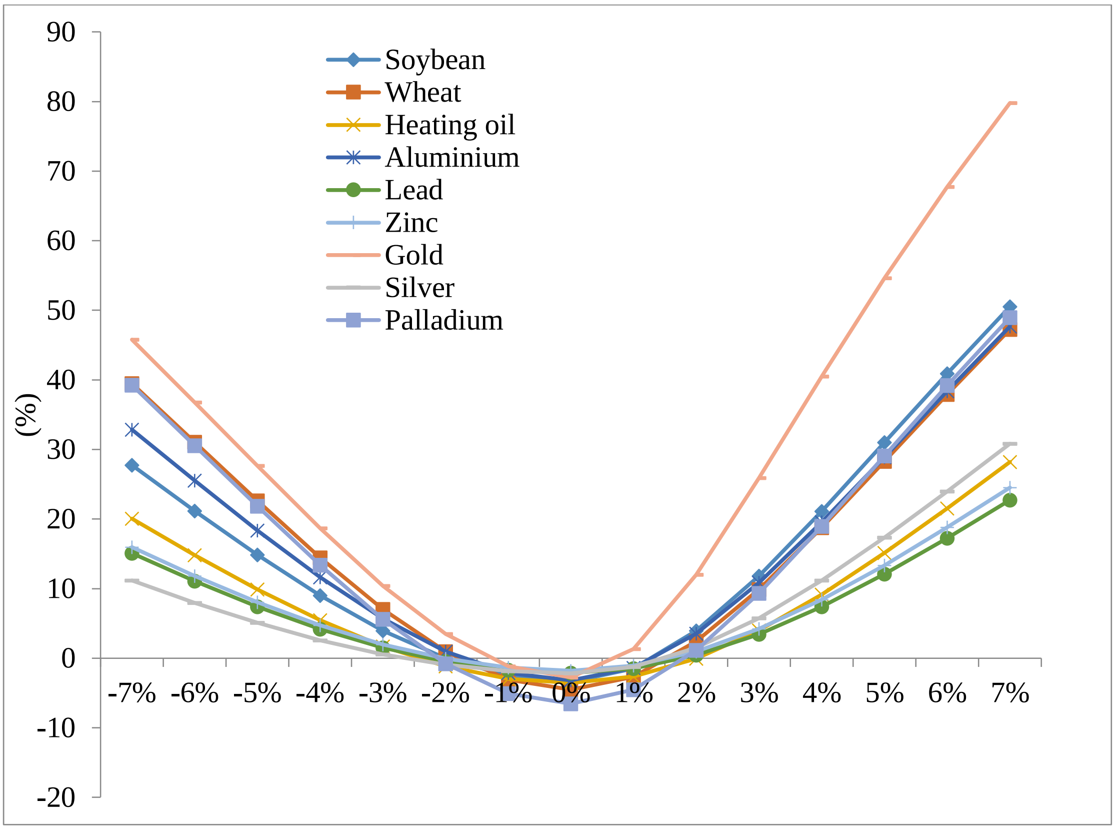


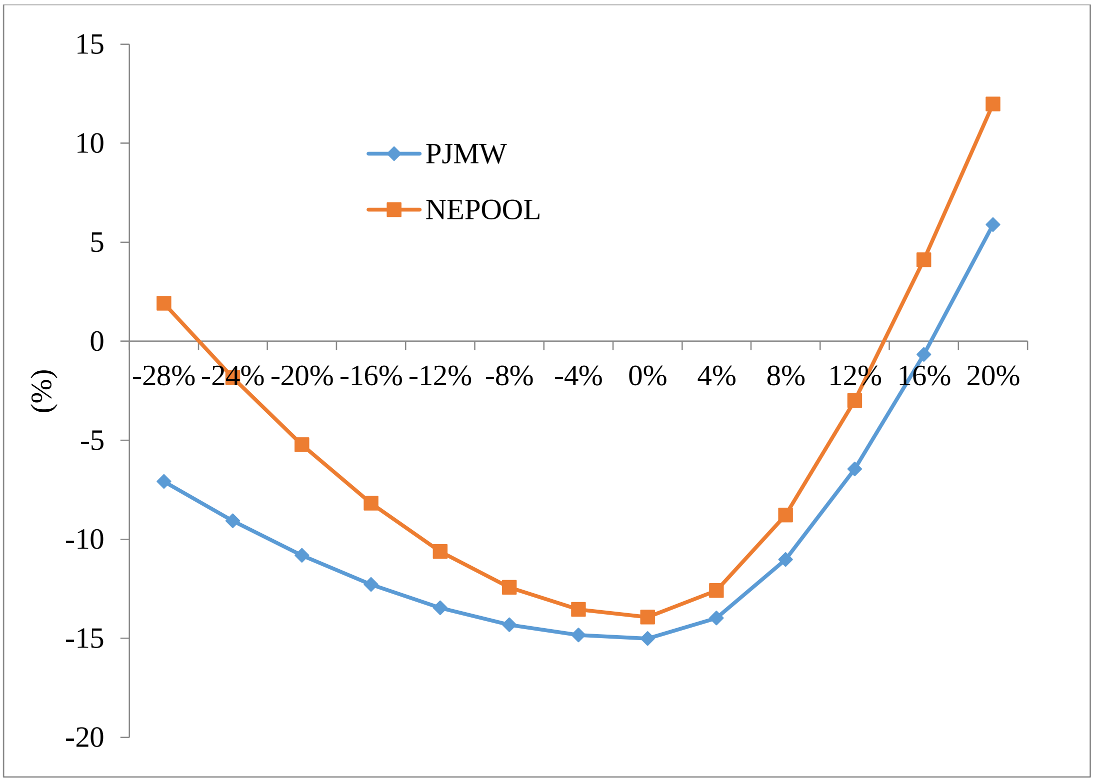
(Figure 1c: $\psi = 1.05, \gamma = 1.5, \sigma = 20\%, C_0 = 1$, and $\epsilon = 1$)

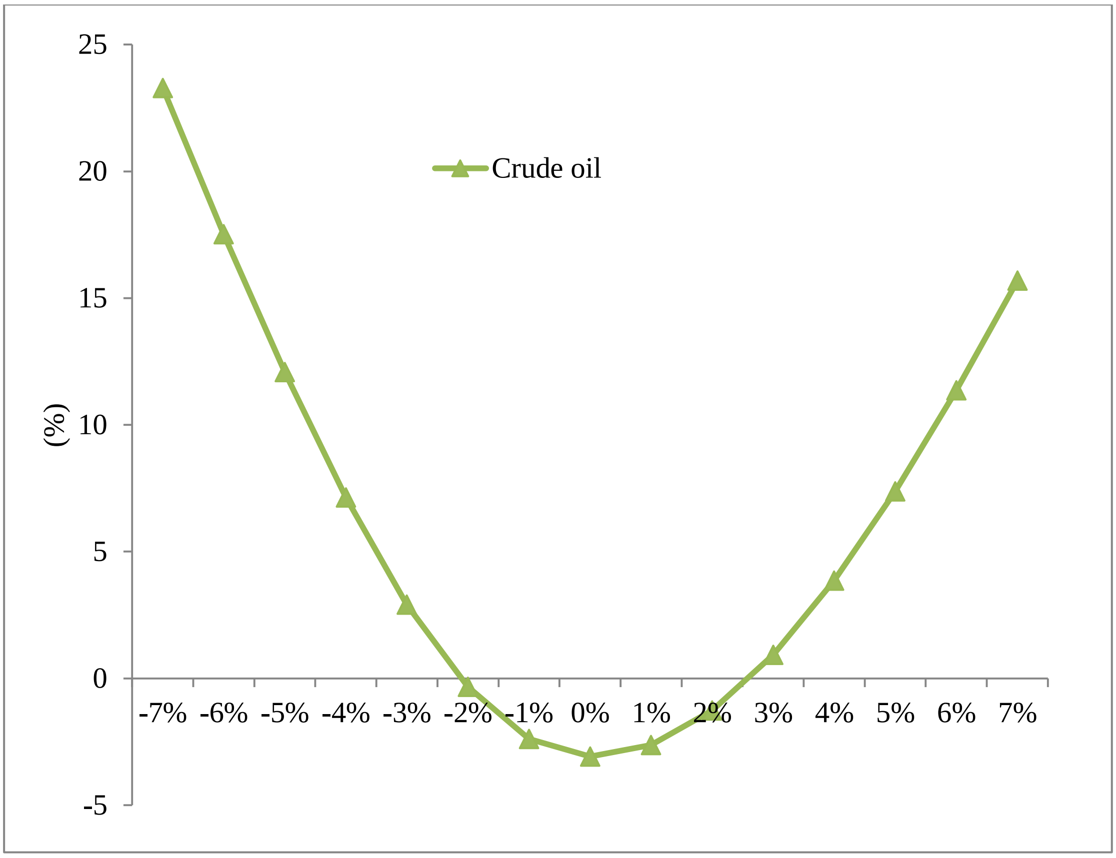


(Figure 1d: $\psi = 1.05, \gamma = 0.9, \sigma = 20\%, C_0 = 1$, and $\epsilon = 1$)

Figure 2 Change in conditional volatility following a return surprise







Notes: This figure depicts the log percentage changes ($\ln\sigma_{t+1}/\sigma_t$) following a surprise market return on day *t* based on the TGARCH estimation reported in Table 2. The volatility before the shock was assumed at its unconditional mean level.

Appendix 1 Price dynamics under mean-reversion

If Z_t follows a mean-reverting process

$$dZ_t = \lambda(\bar{Z} - Z_t)dt + \sigma Z_t dW_t, \tag{A.1}$$

where $\bar{Z}$ is the long-run mean the demand would return to, and λ denotes the parameter related to adjustment speed. Apply Ito's Lemma to equation (4), we obtain similar result as in equation (6)

$$dp_t = \left(\frac{\alpha\epsilon^2\sigma^2 Z_t^2}{2(1+\epsilon Z_t)^2} - \frac{\alpha\epsilon\lambda(\bar{Z} - Z_t)}{(1+\epsilon Z_t)} + \frac{(1+\alpha)\lambda(\bar{Z} - Z_t)}{Z_t} - \frac{1}{2}(1+\alpha)\sigma^2\right)p_t dt$$

$$+ \left(1 + \frac{\alpha}{(1+\epsilon Z_t)}\right)\sigma p_t dW_t \tag{A.2}$$

where $\alpha = \frac{\frac{1-\psi}{\psi}}{\left(\gamma - \frac{1}{\psi}\right)}$.

Appendix 2 Data Description

Commodity	Description	Pricing unit
	Agricultural products	
Corn	Corn No.2 Yellow	¢/bushel
Soybean	Soybeans, No.1 Yellow	¢/bushel
Wheat	Wheat, No.2 Hard (Kansas)	¢/bushel
	Energy products	
Crude oil	Light-Sweet crude futures, Cushing, Oklahoma	$/barrel
Gasoline	New York Harbor Reformulated Regular Gasoline futures	$/gallon
Heating oil	New York Harbor No. 2 Heating Oil futures	$/gallon
Natural gas	Natural Gas Futures, Henry Hub	$/MMBtu
Electricity (NEPOOL)	NEPOOL (New England power pool) day-ahead price	$/MWh
Electricity (PJMW)	PJM West hub peak price (Pennsylvania-New Jersey-Maryland power pool)	$/MWh
	Industrial metals	
Aluminium	LME-Aluminium 99.7% Cash & 3-month futures	$/metric ton
Copper	LME-Copper, Grade A Cash & 3-month futures	$/metric ton
Lead	LME-Lead Cash & 3-month futures	$/metric ton
Nickel	LME-Nickel Cash & 3-month futures	$/metric ton
Tin	LME-SHG Zinc 99.995% Cash & 3-month futures	$/metric ton
Zinc	LME-Tin 99.85% Cash & 3-month futures	$/metric ton
	Precious metals	
Gold	Gold Bullion LBM	$/Troy Ounce
Silver	Silver Fix LBM Cash	¢/Troy Ounce
Platinum	Platinum,Industrial (Engelhard)	$/Troy Ounce
Palladium	Palladium	$/Troy Ounce

Note: The electricity price for NEPOOL was the Mass hub day-ahead locational marginal price.