The Impact of U.S. Shale Oil on the Global Oil Market: Case Study, Bakken Shale

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Abstract

We examine production from horizontal wells in the Bakken Shale since April of 2015. There is strong evidence of rapid and continuing technological progress, geographic heterogeneity within the play, interference across wells that diminishes production, and that this source of supply is substantially more price-elastic in the short term than non-OPEC supply as a whole. We also find the familiar rapid early decline rates characteristic of shale oil, but that production continues at some level long after these early declines. On balance, these results support the view that shale oil has a significant stabilizing effect on world oil markets.

Keywords: Bakken; horizontal; technology; elasticity

JEL Classification: Q410

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Introduction

The rapid development of shale oil in the U.S. has had a significant impact on the pricing and availability of global oil supplies. The abundance of the resource surprised the industry and may be largely responsible for the moderate prices observed since 2014. It has also turned the U.S., once again, into an energy powerhouse; now vying to be the largest oil producer in the world. Nonetheless, the resource’s development has raised a series of interesting questions:

1. As the “marginal resource,” how important has shale oil become in balancing global oil demand and supply?
2. Rapidly falling oil prices in 2015 and 2016 caused drilling and well completion to drop significantly, and yet, surprisingly, production held firm, despite shale oil’s accelerated decline rate. How can this be explained?
3. Although shale oil declines rapidly in the first few months, it also has a “long tail.” 97% of all the horizontal wells drilled in North Dakota are still producing oil. How will this impact the future oil market?
4. Any natural resource has an earth-bound limit and yet the shale resource now seems unbounded. How long before “peak oil,” again, emerges as a concern? Put another way, is shale oil a passing fad or a permanent reset of the oil market?

We do not pretend to have full and complete answers to these questions. Nonetheless, an in-depth review of one of the major shale plays in the U.S. provides some insight into the significance of shale oil development with respect to the global oil market.

Background on Bakken oil development

The Bakken shale, which has been a major source of U.S. shale oil development, is centered in North Dakota. Annual oil production in North Dakota increased from 123 thousand barrels per day in 2007, to 1.28 million barrels per day at the end of 2018, a ten-fold increase. The state provides publicly available data on the number and types of oil wells, producers in the state, well locations, monthly production, and other pertinent data. As of 2018, North Dakota listed 17,328 active oil wells; approximately one-half are horizontal wells from the Bakken formation. In 2018 these wells accounted for 58% of the state’s oil production. The modeling effort described later on focuses on the 2,088 horizontal wells in the formation that were completed after March 2015.

Bakken oil development has been remarkably diverse; in North Dakota there are 137 producers and the vast majority are small independent oil companies. Continental

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Resources is the largest producer, but produces only 11.7% of the crude oil. The region has been plagued by infrastructure shortages. Monthly well production varies so dramatically that traditional gathering pipelines are frequently impractical and trucking is the main way to move the oil. Pipeline capacity out of the state is limited, leaving much of the interstate transport to oil trains.

The diverse and constrained infrastructure has had an impact on pricing. During periods of rapid growth (2012 to 2014) North Dakota “first purchase” oil prices trailed West Texas Intermediate (WTI) prices by as much as $13 per barrel, even as WTI itself was discounted. In 2018 the discount below WTI averaged $4.31 per barrel, but with substantial variation. Generally, the high cost of moving Bakken oil to market explains the differential, but companies with access to interstate pipelines receive a much higher netback.

**Shale oil as balance wheel**

Conventional oil fields have a long development lead time and production flow that can last for decades. For example, Prudhoe Bay was discovered in 1968, but the oil did not come to market until 1977 following the completion of the Alyesaka Pipeline. North Slope oil production peaked over a decade later in 1989 and has declined at a slow rate since. In contrast, shale oil has a short development time. Moreover, production declines rapidly: nearly half of a horizontal well’s total production will be produced in the first 12 months. Consequently, shale oil is closer to conventional manufacturing and is best explained by the Marshalian partial equilibrium model of a firm. In the competitive model, flexible production along with rapid entry and exit stabilizes the market and makes prices more predictable. In contrast, conventional oil has exhibited long-term price cycles. Prices have peaked in periods of constrained capacity, then, fallen back for decades, as consumers retrench. Given the lead times and massive capital investment of conventional oil fields, prices match long-term marginal costs only by accident.

OPEC depends on a swing producer or producers to stabilize prices. Given pressure from other oil exporters, however, the cartel’s largest producer, Saudi Arabia, has found this roll increasingly difficult and resists going it alone. In most circumstances the margin of oil supply that balances the market is thought to be small, on the order of 3 to 5 million barrels per day or less out of global demand of just over 98 million barrels per day. In contrast, tight oil production in the U.S. alone has now reached 7.5 million barrels per day. If well development were to stop cold this figure could drop by half in little over a year. Likewise, given adequate incentives, production can ramp up quickly. In 2018, Bakken oil output increased 28%, despite relatively modest oil prices.

The flexibility of the shale resource is measured by price elasticity. Shale oil’s supply short-run price-elasticity is much higher than other non-OPEC sources. The econometric model (described later in the paper) estimates Bakken horizontal well short-run price
elasticity at +0.2636. The long-run elasticity, +0.6453, is the total response to a permanent change in price, and most of that response occurs within the first year. In contrast, estimates of conventional crude oil supply price elasticity are much lower, especially in the short term: Vatter (2017) estimated within-quarter elasticity of non-OPEC supply to be 0.015.\(^5\) Golombek et al. (2018) noted: “There are not many estimates of the non-OPEC supply elasticity in the literature.” They estimated the long-run price elasticity of non-OPEC supply to be 0.32, and they cite Alhajji and Huettner (2000), who estimated it to be 0.29.\(^6\) Vatter estimated it to be 0.24.\(^7\)

Indeed, it is the contrast between the short-run price elasticity of shale oil compared to conventional oil fields that allows shale oil to play its balancing role. Shale oils and conventional oils are substitutes, effectively identical once produced. Conventional oil fields are generally constrained by the natural flow of the oil. Shale oil fracking induces a spurt of production, one that with relative ease (at a given cost) can be turned off and on. As a consequence, the range of high and low prices is likely to be smaller than it has been in the past and, ultimately, oil prices will tend toward the long-run marginal cost of the shale oil resource, similar to the role now played by shale gas in North America. It is also worth noting that shale oils compliment inventory management. Previously the lead times of conventional oil development were too long to offset shortages with inventory drawdown or to ramp up storage during a glut. Most futures trading concentrates in the first two years of forward schedules. As such, it fits nicely with the shale oil production cycle.

**Impact of the drop in oil prices**

Since 2014, oil prices have remained low and volatile. Lower prices have had a direct impact on drilling and the number of wells completed. Drilling statistics can be misleading because there is a lag that varies between drilling and well completion. According to the EIA, wells drilled in the Bakken formation (including Montana) dropped from 250 in June 2014 to 43 in June 2016.\(^8\) The drop in drilling, however, had less of an impact than the figures might imply, because there were 53 well completions, offsetting some of the expected decline from less drilling. Misuse of drilling data alone can lead to misunderstandings about well productivity. Fortunately, North Dakota keeps data on well completion specific to horizontal wells. Figure 1 illustrates the relationship

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\(^7\) Ibid, Vatter.

\(^8\) EIA, [https://www.eia.gov/petroleum/drilling/](https://www.eia.gov/petroleum/drilling/) accessed March 19, 2019.
between oil prices and the number of North Dakota horizontal wells in the Bakken formation that were completed.

It is immediately evident that the collapse of the oil market in 2014 caused a precipitous drop in well completion. From 2014 to 2016 completions of horizontal wells fell 64% in North Dakota, while oil production in the state fell only 5%. There are several reasons for this startling result. First, as noted, not all the production is from horizontal wells; conventional production is mixed in. Even so, however, the extraordinary feature of the three-year period has been a significant increase in well productivity. The first full month of production from a horizontal well completed in September 2018 averaged 63% more than one completed in September 2015. This is an astonishing technological gain.
There are a number of reasons given for the technological shift: improved resource knowledge, a longer drilling range, better fracking techniques, improved chemicals, etc. Figure 2, based on Equation (1), as explained in the modeling section, illustrates the shift in productivity and the rapid decline rate of shale oil in the first months of production. The chart also projects forward 75 months, for wells going into production in August of 2015 and August of 2018. Figure 2 assumes that price, location, and distance between wells are fixed in order to isolate the effect of technological change. With location fixed, inasmuch as learning has related to where to drill, Figure 2 understates the shift. The data support a model in which the rate of decline did not change between 2015 and 2018; instead, there has been a proportional upward shift. In the first year (12 months) of production, output declined 79% for both 2015 and 2018 wells.

Long tail of production

The rapid decline in production the first year after fracking does not continue indefinitely. In fact, horizontal wells in the Bakken have substantial longevity. This seeming contradiction is a consequence of the well going from transient flow to boundary-dominate flow (or some other flow regime), as described in Attanasi et al. (2019). The full history of Bakken horizontal wells presented in Table 1 (next page) demonstrates this revealing characteristic. The decade from 1986 to 1995 appears to have been primarily a period of experimentation. Low oil prices effectively ended the experiment until 2004, when the market turned around. Overall, 97.1% of horizontal wells drilled are still in production. Following the initial spurt of production, wells produce only around 10% of their peak, but they continue to produce for a long time.

Figure 3 is a scatter diagram comparing the date of well completion to production in November 2018. The chart illustrates both the relationship, as well as substantial variation in production rates.
Diminishing returns

Analysis of horizontal well production data has revealed significant technological progress over a three-year period; so much so that it has offset a major production decline that would have been expected following the drop in drilling. Nonetheless, oil is a depleting resource and at some point there must be diminishing returns. The econometric model provides some insight about this point.

Bakken shale is the source rock that supported historic oil production from the Williston Basin. It is located in the northwestern one-third of North Dakota, centered in the Fort Berthold Reservation. Table 2 shows the relative productivity of wells by geographic location. The modeling results suggest a range of over 5 to 1. In other words, when
drilling moves from the most productive center of the formation to the periphery, productivity drops substantially.

Recently, it has been suggested that well spacing is another factor in productivity.\(^9\) Using the Public Land Survey System, the model included data for the number of wells in each quarter-quarter section in order to estimate the impact of well spacing. One additional well in the same quarter-quarter section is associated with a 5% decline in production. This suggests that while the Bakken play may be experiencing some diminishing returns, technological improvements have more than overcome it.

Assessing shale oil’s future potential starts with the recognition that, so far, development has been constrained to regions with historic oil production, such as the Williston Basin. This may be due to the industry’s familiarity with historic producing areas, but most likely it is simply geology. Figure 4 illustrates the various sources of shale oil in the U.S. Virtually all of the production is located in well-known oil regions. Indeed, 89% of shale oil production is from source rock supporting the Permian, Bakken, and East Texas historic plays. While shale oil has proven to be highly prolific, it is not geographically dispersed.

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The pattern of development in the U.S. hints at the future for oil in the rest of the globe. OPEC’s control of the oil market since 1973 follows from the concentration of low-cost resources in the Middle East. Conventional oil production, like shale oil in the U.S., is not geographically dispersed. Consequently, it is likely that future global shale oil development will be limited to the regions already rich in the resource. As the saying goes, “plus ça change, plus c'est la même chose.”

Description of the model

Table 3 summarizes the panel data used in the analysis.

Table 3: Summary of data by well and month

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production (bbl)</td>
<td>7,188</td>
<td>7,781</td>
<td>0</td>
<td>81,686</td>
</tr>
<tr>
<td>Months in production</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>7.75</td>
<td>10.83</td>
<td>0</td>
<td>45</td>
</tr>
<tr>
<td>Beginning with peak</td>
<td>6.84</td>
<td>10.22</td>
<td>0</td>
<td>44</td>
</tr>
<tr>
<td>Initial month (April 2015 = 0)</td>
<td>21.34</td>
<td>13.79</td>
<td>0</td>
<td>44</td>
</tr>
<tr>
<td>Co-located wells</td>
<td>1.24</td>
<td>1.48</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>Price (Jan 2019$/bbl)</td>
<td>48.31</td>
<td>10.91</td>
<td>24.12</td>
<td>67.77</td>
</tr>
<tr>
<td>Latitude</td>
<td>47.96</td>
<td>0.30</td>
<td>46.92</td>
<td>48.99</td>
</tr>
<tr>
<td>Longitude</td>
<td>-102.98</td>
<td>0.41</td>
<td>-103.99</td>
<td>-102.12</td>
</tr>
</tbody>
</table>

The following equation is estimated assuming a different randomly distributed intercept for each well:
where,

\[ q_{it} = 40.0987 - 0.0024 M_{it} - 0.3994 m_{it} + 0.3521 q_{it-1} - 0.0512 W_{it-1} + 0.0101 I_i - 124.592 e^{Long_i} - 5.5842 Lat_i - 9.1460 Long_i + 12.3971 \ln Lat_i - 50.2772 \ln (\text{Long}_i) - 0.3974 \text{Lat}_i \text{gLong}_i + 0.1151 dp_i \]  

(1)

\[ \text{where,} \]

\[ q_{it} = \text{log production in Well } i \text{ in Month } t \text{ in bbl} \]

\[ M_{it} = \text{months in production starting with peak at Well } i \text{ as of Month } t \]

\[ m_{it} = \text{log months in production starting with peak at Well } i \text{ as of Month } t \]

\[ W_{it} = \text{number of producing wells in the same PLSS QQ section as Well } i \text{ as of Month } t \]

\[ I_i = \text{initial month of production at Well } i \]

\[ \text{Long}_i = \text{longitude of Well } i, +100^\circ \]

\[ \text{Lat}_i = \text{latitude of Well } i, -45^\circ \]

\[ dp_i = \text{first difference in log North Dakota first purchase price in Month } t \text{ in } \$/$bbl, adjusted for inflation using the Consumer Price Index

We use a random-effects estimator because latitude and longitude do not vary over time. For inference, the standard errors were adjusted for clustering on well.

Equation (1) was estimated in two stages because the first month in production is generally not a full month of production. As a result, after estimating \( q_{it} \) with \( t = 1 \) the second month of production, the lagged dependent variable, \( q_{it-1} \), would not reflect a full month of production. Therefore, the regression was run beginning with \( t = 3 \) the third month in production to estimate the coefficient on \( q_{it-1} \left( \frac{0.3521}{0.0100} \right) \), and, holding that coefficient constant, the other coefficients were estimated. That is, \( q_{it} - 0.3521 q_{it-1} \) was regressed on the other variables using data beginning in the second month of production. This is not perfect, because observations in the first month still have some effect. However, it is important to include observations of production in the second month, which is the first full month of production, in the dependent variable, as they are crucial to estimating the steep early declines of shale oil production.

The log of production was used as the dependent variable so that predicted production will never be negative. This also has the effect of ignoring observations with zero production. Equation (1) assumes wells are continuously in production, ignoring gaps.
This is valid for estimating a continuous decline curve, but a different assumption, below, was used when estimating the price elasticity of supply. We have also excluded observations of positive production below 100 barrels per month, as those data appear to be erroneous.

All of the coefficients are highly statistically significant, except the level of longitude, which is significant at the 99% level. The full Stata header is shown in Table 5.

The overall \( R \)-squared depends on whether the explanatory power of persistence is included. In the regression beginning with the third month in production, where the effect of \( q_{it-1} \) is estimated, the overall \( R \)-squared is 0.66. Holding the effect of \( q_{it-1} \) constant, the overall \( R \)-squared measures the fraction of the variation in production explained by the remaining variables, and that is 0.42. There is a little more variation to explain with data in the first full month of production included, as the coefficient of variation in production \( \rho \), the fraction of the variance in \( q_{it} \) explained by the random effects, is 0.04 when estimating the coefficient on \( q_{it-1} \), and 0.24 when the variation associated with \( q_{it-1} \) is removed.

Both levels and logs of months in production since peak have negative coefficients, and this functional flexibility allows for the characteristic rapid initial decline rates of shale wells. We assumed that a well was in production during a given month if production exceeded 100 bbl.

The estimated persistence in production, 0.3521, suggests that operators have broad flexibility to vary production from one month to the next, but that some such changes would be difficult or impose a cost. Since the sample omits gaps in production, the coefficient on \( q_{it-1} \) does not reflect the cost of shutting in or restarting a well. In estimating persistence when also estimating the price elasticity of supply, gaps in production were allowed.

The number of co-located wells that are in production bears a negative coefficient. If wells are not adequately spaced, production is adversely affected. A co-located well was counted as being “in production” if it produced more than 100 barrels. To guard against endogeneity of the regressor, there is a 1-month lag in the number of co-located wells, \( W_{it-1} \).

The coefficient on initial month of production, \( I_i \), is positive, reflecting technological progress. By specifying initial month in levels and production in logs, the impact of technological progress is greatest when wells are at their most productive, early in their lives. Operators are often simply experimenting as they go, and, as Kah (2018) wrote: “While improvement from any specific activity may come to an end, there should still be
a long way to go in overall technology advancement. Hence, a simple function of time is used to measure technological progress.

A very general function of latitude and longitude was specified to describe location in order to allow for nuances in its relationship to production. The function originally included exponents, levels, and logs of latitude and longitude, as well as the product of the two. Only the exponent of latitude was dropped from Equation (1) because it was not statistically significant. Dummy variables for township, range, or section, were considered, but the precision of latitude and longitude explained production as well as any of the dummies, with fewer regressors. 45° was subtracted from the values of latitude and 100° added to the values of longitude so that they would exhibit some significant variation in relation to their means, and Stata would not drop them for being collinear.

Figure 5 shows the relationship between location and production after controlling for the other variables in Equation (1). The “sweet spot” is at 47.7463° latitude and -102.6929° longitude.

The log of price was used so that the supply curve would be convex. The first difference was used because price is a strongly trending variable: The correlation between log price and time is 0.67, while the correlation between the first difference and time is essentially zero. In Equation (1) the coefficient on the contemporaneous first difference

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in log price, $dp_t$, is positive. Since operators can always change the rate of flow on a well the result makes sense. With the lagged dependent variable included, the impacts on production of a change in price lasting $s$ months increases over time as the sum of a geometric series.\(^{11}\)

\[
dq_{it} = 0.1151 dp_{t-s} \sum_{j=0}^{t-s} 0.3521^j
\]

\[
= 0.1151 dp_{t-s} \left( \frac{1 - 0.3521^{t-s+1}}{1 - 0.3521} \right)
\]

\[
\rightarrow \frac{0.1151 dp_{t-s}}{1 - 0.3521} \text{ as } t \to \infty
\]

With more lead time, operators can do more to change the rate of flow from a well. However, the data used to estimate Equation (1) reflect an assumption that wells are continuously in production, and this restriction excludes variation in production associated with shutting in and restarting wells, which, to some extent, reflects consideration of price.

For two selected months, the model predicts production in the second month between the median and the mean, within sample, as shown in Table 4. Here, location is fixed at 48.3915° latitude and -102.7500° longitude. For all but five months in the sample, median production for wells in the second month of production is below the mean, and the average ratio of mean to median is 1.11.

<table>
<thead>
<tr>
<th></th>
<th>Median</th>
<th>Prediction</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sep-18</td>
<td>19,871</td>
<td>20,857</td>
<td>23,612</td>
</tr>
<tr>
<td>Sep-15</td>
<td>12,087</td>
<td>13,855</td>
<td>14,449</td>
</tr>
</tbody>
</table>

To predict production in the first month using Equation (1), we let $t - 1$ approach $t$ and set $M_i = 1$, which gives us the following equation:

\[
q_i = 61.8897 - 0.0790W_i \\
+ 0.0156I_i - 192.3113e^{Long_i} - 8.6194Lat_i - 14.1170Long_i \\
+ 19.1352 \ln Lat_i - 77.6043 \ln (-Long_i) - 0.6134Lat_i gLong_i
\]  

In the sample, 653 wells peak in the first month, 675 in the second, 320 in the third, 191 in the fourth, and 100 in the fifth. It is safe to say that the first month, if it were a full month of production, would be the modal peak month. This is why the first month was modeled as the peak month.

**Price Elasticity of Supply**

Ignoring gaps in production is benign for the purpose of estimating a continuous production curve like that shown in Figure 2, but gaps in production, when they begin, when they continue, and when they end, reflect, to some extent, consideration of price, even if that is not the main consideration. To estimate the effect of price on production, the elasticity of supply, production was assumed to be 1 barrel during months when production was actually zero, so that the dependent variable, the log of production, would be defined. The variation between 1 barrel and observations over 100 barrels is virtually the same as between 0 barrels and the same observations over 100 bbl. With the modified data, the coefficient on the level of months in production beginning with peak does not take the expected sign, so it was dropped from the regression. (The implication of a positive coefficient on \( M_{it} \) would be an eventual and perpetual increase in production from a given well.) The exponent of latitude is significant, so we include it. Equation (3) was estimated using the same two-step procedure used to estimate Equation (1).

\[
q_{it} = 27.7185 - 0.3042m_{it} + 0.5915q_{it-1} - 0.0605W_{it-1} \\
+ 0.0066I_{it} - 0.0657e^{Lat_{it}} - 90.6795e^{Long_{it}} + 2.1920Lat_{it} \\
- 5.9368\ln Lat_{it} - 38.8940\ln (-Long_{it}) - 0.3853Lat_{it} gLong_{it} \\
+ 0.2636dp_{it} 
\]  

Persistence, the coefficient on \( q_{it-1} \), 0.5915, is considerably larger than in Equation (1), reflecting the greater costs associated with shutting in and restarting wells, as compared to those of adjusting continuous production. The coefficient on price, 0.2636, is also considerably larger than in Equation (1), reflecting the greater variation in production being observed when gaps in production are included in the sample. Notably, however, the overall R-squared when gaps are included in the sample (and the coefficient on the lagged dependent variable is being estimated) is 0.46, as compared to 0.66 when gaps
are omitted. Most of the reasons for shutting in and restarting wells are independent of price.

In estimating the total contribution of shale oil to the elasticity of non-OPEC supply, however, it is important to look at the variation in production represented by gaps. With gaps included, the estimated short-run price elasticity of supply is 0.2636, with a 95% confidence interval of \([0.1343, 0.3929]\), and the estimated long-run price elasticity of supply is \(0.2636/(1 - 0.5915) = 0.6453\), with a 95% confidence interval of \([0.3183, 0.9722]\). These compare to 0.1151 in the short-run and 0.1776 in the long-run when gaps are omitted.

Our estimate of supply in the Bakken is elastic compared to non-OPEC production in general. Shale wells are more labor- and materials-intensive than conventional wells, and variable costs are higher, so this result is not surprising. It is said that the Bakken is the most price-sensitive of the shale plays\(^{12}\), but, inasmuch as the greater elasticity of the Bakken is indicative of the supply of shale oil in general, it is good news for the world’s consumers and the world economy, as greater elasticity of supply will dampen volatility in price and reduce OPEC’s market power. However, shale production may not fully rescue the consumer and the economy from the volatility in oil prices they have known since 1973.\(^{13}\)

**Conclusion**

We conclude that technological progress in the Bakken Shale and, by extension, other shale plays, is rapid and continuing, and that this source of supply is more price-elastic than non-OPEC supply in general. Accordingly, shale drilling has a significant stabilizing influence on world oil markets. This view is mitigated by evidence of diminished production caused by interference from nearby wells.


Table 5: Stata header for estimation with continuous production

<table>
<thead>
<tr>
<th>Random-effects GLS regression</th>
<th>Number of obs = 39467</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group variable: Well</td>
<td>Number of groups = 1994</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>R-sq: within</th>
<th>w/ q L1.</th>
<th>w/o q L1.</th>
<th>Obs per group:</th>
<th>Wald chi2(11) = 25466.55</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5272</td>
<td>0.3468</td>
<td>min = 1</td>
<td>Prob &gt; chi2 = 0</td>
</tr>
<tr>
<td>between =</td>
<td>0.8606</td>
<td>0.6838</td>
<td>avg = 19.8</td>
<td></td>
</tr>
<tr>
<td>overall =</td>
<td>0.6632</td>
<td>0.4248</td>
<td>max = 43</td>
<td></td>
</tr>
<tr>
<td>rho =</td>
<td>0.0407</td>
<td>0.2373</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Random effects $u_i \sim$ Gaussian
corr($u_i$, $X$) = 0 (assumed)

<table>
<thead>
<tr>
<th>Robust</th>
<th>[95% Conf. Interval]</th>
</tr>
</thead>
</table>

| q      | Coef. | Std. Err. | z     | P>|z| | [95% | Interval | |
|--------|-------|-----------|-------|------|-------|---------|-------|
| _cons  | 40.0987 | 7.0416   | 5.69  | 0     | 26.2974 | 53.9000 |
| Mpeak  | -0.0024 | 0.0006   | -4.38 | 0     | -0.0035 | -0.0013 |
| mpeak  | -0.3994 | 0.0064   | -62.08| 0     | -0.4120 | -0.3868 |
| q L1.   | 0.3521  | 0.0100   | 35.22 | 0     | 0.3325  | 0.3717  |
| CWL L1. | -0.0512 | 0.0060   | -8.6  | 0     | -0.0628 | -0.0395 |
| Initial | 0.0101  | 0.0006   | 17.76 | 0     | 0.0090  | 0.0112  |
| expsLong| -124.5920 | 28.7955  | -4.33 | 0     | -181.0300 | -68.1539 |
| sLat   | -5.5842  | 0.3899   | -14.32| 0     | -6.3483 | -4.8201 |
| sLong  | -9.1460  | 3.0153   | -3.03 | 0.002 | -15.0558 | -3.2362 |
| slat   | 12.3971  | 0.7660   | 16.18 | 0     | 10.8958 | 13.8984 |
| snlong | -50.2772 | 13.2967  | -3.78 | 0     | -76.3382 | -24.2162 |
| sLL    | -0.3974  | 0.0622   | -6.39 | 0     | -0.5194 | -0.2754 |
| dp     | 0.1151   | 0.0257   | 4.48  | 0     | 0.0647  | 0.1654  |
References


