Are Complex Energy System Models More Accurate? An Intra-Model Comparison of Power System Optimization Models

Jan Priesmann[1a], Lars Nolting[a], Aaron Praktiknjo[1a]

[a] *RWTH Aachen University, Institute for Future Energy Consumer Needs and Behavior (FCN), Mathieustr. 10, 52074 Aachen, Germany*

Abstract

Today, optimization models are by far the most popular choice when analyzing energy systems. Impressive advances in computer and data sciences have allowed for a multitude of complex energy system optimization models. The goal of our work is to assess the hopes of a positive relationship between the complexity of the model and the accuracy of the results. Up until now, a benchmarking of different complexity levels has only been performed for individual system components such as for the operational behavior of power plants, the transmission grid or the temporal resolution. We propose a framework based on alternative model formulations and apply it in a case study with 160 different, more or less complex implementations of power system optimization models for economic dispatch and investment decisions. Our results indicate that a certain degree of complexity is necessary for sufficiently accurate results, however, a careful balancing is required for an efficient use of computational resources. We find that most of the Pareto optimal implementations of dispatch models show temporal complexity (i.e. solving time) below 1.2% and spatial complexity (i.e. memory usage) below 0.3% of the respective maximum complexity observed. We conclude that different formulations for the partial load efficiency of conversion processes can be recommended for each of the two decision problems we analyzed. We further find that a simple grid model comes with a minor increase in complexity compared to a copper plate model. The methodology developed shows to be promising in reducing computational effort and in providing practical guidance for model developers.

Keywords

Energy system optimization models; Mixed integer programming; Complexity reduction; Pareto optimality; Power supply system

1 Introduction

The systematic analysis of energy systems using energy system models has been recently affected by three significant developments. First, the ongoing energy transition brings rapid and continuous change to energy supply, demand, and policies under a framework of ecological, economic and security of supply constraints [1]. Depicting this increasing complexity represents a major challenge for the investigation and planning of energy systems [2]. Second, optimization models have become the most popular approach for economic dispatch planning [3], the evaluation of future energy scenarios [4], the assessment of policy measures, [5] and other tasks. Finally, advances in information and data sciences are enabling energy system models to depict an increasing part of the complexity of real energy systems. The attraction of these new possibilities can encourage applying a degree of model complexity that is independent of the problem being addressed [6]. In total, these three developments can be observed through an increase in complex energy system optimization models (ESOMs).

The attractiveness of optimization models lies in their capability of finding a cost optimal solution for energy systems under various constraints [7].They are commonly used for short-term dispatch planning and for investigating future scenarios in terms of investment planning [8]. Constraints can depict physical necessities, such as the balancing of supply and demand, as well as political, social, or environmental constraints, such as limiting greenhouse gas

[1] Corresponding authors, contact: [Jan.Priesmann, APraktiknjo]@eonerc.rwth-aachen.de

emissions [6]. Structural changes in energy systems forced ESOMs to become more complex [2]. A liberalized energy supply system comes with more market actors and more interfaces between them. A distributed energy system comes with an increase in the number of nodes and vertices and therefore the need for higher spatial resolution. Highly volatile renewable energy sources require a higher temporal resolution. Fortunately, due to the progress in computational resources such large ESOMs have become solvable even on desktop computers [2]. Today's ESOMs usually combine expansion and dispatch planning, enabling the investigation of highly distributed and volatile future energy systems. With efficient mixed-integer linear programming (MILP) solvers, individual units can be depicted with high level of detail [9]. State-of-the-art ESOMs, however, are still restricted by the limits of computing resources as they become more and more complex [10].

According to George Box and Norman Draper, 'all models are wrong, but some are useful' (cited by [11]). If models are wrong in general but still can be a useful representation of reality, the question arises as to how simple an ESOM can be while retaining the required accuracy in representing the real system [12]. In general, models should be kept as simple as possible and as complex as necessary to use resources efficiently and reach the goal of parsimony [13]. Choosing the model with the best trade-off between complexity and accuracy comes with a process that starts with (1) the formulation of a research question, and includes choosing (2) the conceptual model, (3) the necessary model components, (4) their relations, and (5) the degree of detail (e.g. in terms of temporal or spatial resolution) which, in combination, are the minimum requirements to answer the research question [14]. The process steps following the formulation of the research question should be the result of an inter- (step 2) and intra-model complexity comparison (steps 3, 4, and 5). Hereby an appropriate conceptual model is selected and further specified in its complexity and degree of detail according to the individual requirements. Such a systematic procedure is uncommon in energy system modeling though it would allow managing the complexity in energy system models by generating a scope of options with different levels of complexity.

Inter-model complexity comparison can be found in different disciplines. García-Callejas and Araújo [15] compare different models for ecological systems for their complexity and their accuracy in representing the real ecologic system behavior. Venkataraman and Haftka [16] analyze different structural models for buildings. Different hydrological models are compared by Orth et al. [17]. Bale et al. [18] compare different models for their capability of representing complex system behavior in energy system. Meta-modeling can be considered as a type of inter-model complexity comparison that replaces the original model by a less complex representation. Ikeda and Ooka [19] apply meta-modeling for optimization models of a building energy system and observe the potential for large reduction of computing times. Martinez-Moyano [20] presents a documentation tool for system dynamics models that includes information on the complexity of the respective models. Finally, Scheller and Bruckner [21] review different ESOMs for their incorporated complexity and degree of detail, however, their recommendations for making the investigated models more complex are not made conditional on a systematic evaluation of complexity.

In terms of intra-model complexity comparison, most studies focus on specific system components that cause complexity problems. The degree of complexity is varied by analyzing different implementations and the resulting accuracy is compared. Lin et al. [22] evaluate different piece-wise linearization methods by comparing the number of variables and constraints to the approximation error. Milan et al. [23] apply two approaches for linearizing partial load efficiencies in investment ESOMs and compare them by the resulting energy system layouts and the complexity measures CPU time as well as the number and type of variables and constraints. Kotzur et al. [24] aggregate time series to typical days using different clustering techniques and compare the solving times to the deviation in objective function value

(OFV). Marquant et al. [25] apply typical days and rolling horizons to reduce the number of time steps in the optimization model and contrast the resulting solving times with deviation in OFV. Palmintier and Webster [9] aggregate generation units by applying different clustering methods and compare the results among other measures in terms of deviation in OFV, dispatch schedule and solving times.

Studies applying holistic and empirical examinations of complexity and accuracy across multiple components within one conceptual model are rare. Pollok and Bender [26] introduce a workflow that uses multi-objective optimization to find a Pareto front for the trade-off between complexity and accuracy (solving time and a custom error measure) and apply it to Modelica[2] models. Sun et al. [28] propose a systematic procedure for defining the right degree of detail in agent based models. However, there are no comprehensive guides that we are aware of that holistically and empirically examine ESOMs in terms of complexity and accuracy by comparing different more or less complex model formulations (i.e. intra-model complexity comparison). Such an examination would make the modeling process of ESOMs more efficient by minimizing the time required for modeling and computation while at the same time providing a sufficiently accurate answer to the problem [29].

In the context of this paper, we present a methodology that systematically analyzes the trade-off between complexity and accuracy in ESOMs. Our empirical analysis investigates the complexity for the case study of power system optimization models (PSOMs) with regard to economic dispatch and investment planning of energy systems. In addition to the costs of required computing capacity and computing time, complexity management in dispatch models is further motivated by the need of temporal efficient solutions to support short-term operational decisions. We develop a modular and scalable PSOM that allows the generation of a wide range of model variants with different degrees of complexity and accuracy. The analysis of these models provides the empirical basis for the complexity and accuracy assessment. The specific research questions that we will address in this paper are:

(1) Are complex power system models more accurate?
(2) What are the complexity and accuracy drivers in power system optimization models?

Section 2 sets out the literature-based theoretical framework regarding complexity in systems and models, techniques for reducing complexity in ESOMs, and approaches to quantify complexity and accuracy. Resting on this framework, in section 3 the procedure for a systematic management of complexity in ESOMs is introduced and the modular and scalable PSOM formulated and validated. Furthermore, representative model formulations as well as complexity and accuracy indicators are selected. Section 4 presents the optimization results, which are subsequently used to discuss recommendations for the modeling process of PSOMs in section 5.

2 Background: The trade-off between complexity and accuracy in energy system optimization models

The definition of complexity in systems is a long-lasting objective in many disciplines related to system theory[3]. Complexity research can be divided into aggregated complexity, deterministic complexity, and algorithmic complexity [30]. Aggregated complexity deals with systems of linked components and studies the overall system behavior. Deterministic complexity is based on chaos and catastrophe theory [31]. Algorithmic complexity deals with the effort required to solving mathematical problems and with the simplest algorithm required to represent system behavior.

[2] For further information on the Modelica modeling language please refer to [27]
[3] For extensive discussions on complexity definitions in different disciplines please refer to [11].

We regard it as necessary to discuss the term complexity in the context of energy systems and energy system modeling. Therefore, we will define the complexity of the observed system, i.e. the real-world energy system, as aggregated complexity that is defined by decision-making agents, physical and social networks, dynamics, self-organization, path-dependency, emergence, co-evolution, as well as learning and adaption (see [18] for a detailed discussion). We will use computational complexity as a synonym for the required effort for solving mathematical problems in energy system models. Extending the definition of computational complexity, we will further distinguish between inter- and intra-model complexity comparisons. Inter-model complexity comparison analyzes different conceptual models that address the same research question while intra-model complexity comparison analyzes the design of a particular conceptual model. This definition is depicted in Figure 1.

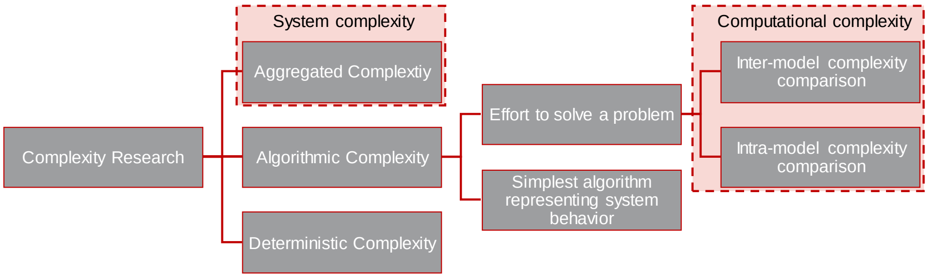

Figure 1: Complexity research framework adapted to computational energy system models (own representation, based on [30])

2.1 Managing complexity in energy system optimization models

Characteristics of complex systems (as found in the definition of aggregated complexity) can also be found among the characteristics of computational complexity in ESOMs. These include: dynamics, i.e. path dependency or the coupling of consecutive time steps; nonlinearities in relations between system components; discrete decisions; and the system size. A representation in the form of a model always simplifies and reduces the complexity that is inherited in the real-world system [32]. This can be observed in a deviation between the three elements of complexity – number of components, connections between components and type of connections [14]. Managing this deviation in ESOMs is based on a range of alternative formulations that offer flexibility in terms of complexity and accuracy.

Based on existing approaches, we divide methodologies for reducing computational complexity into (1) omission or major simplification of system components and relations, (2) approximation techniques, and (3) reformulation and decomposition techniques. We order these three categories on the basis of the associated changes to the original formulation, from significant changes (omitting system components or relations), through slight to medium changes (linear approximation techniques), to the retention of original complexity and degree of detail (reformulation and decomposition techniques). Table 1 gives a non-exhaustive overview of techniques for reducing the complexity in ESOMs that is partly based on [33].

Complexity reduction technique	**Applied change**	**References**	**Targeted complexity problem**
Omission or major simplification	Omission	[34], [35]	Partial load dependent efficiencies: Avoid nonlinearities by assuming constant relations System size: Reduce scope of system boundaries
Linearization	Approximation	[22], [23], [36]	Partial load dependent efficiencies: Avoid nonlinearities by assuming linear relations or discrete steps Economies of scale: Avoiding nonlinearities by assuming linear relations or discrete steps
Temporal aggregation	Approximation	[24], [37]–[39]	Model size: Reducing number of time steps by aggregating consecutive steps or by defining typical days
Technological aggregation	Approximation	[9], [40]–[42]	Model size: Reducing the number of components depicted in the model by aggregating similar components to groups
Reformulation	Accurate	[36], [43], [44]	Exploiting solver behavior: Finding equally exact but more efficient formulations

			Operational properties: Avoiding non-continuous nonlinear relations by using e.g. Big-M reformulation
Model decomposition	(Mostly) Accurate	[45]–[47]	Non-parallelizability of optimization problems: Decompose problem by using model interfaces based on real-world system relations and solve the sub-problems either iteratively (e.g. Benders decomposition) or non-iteratively
Time horizon decomposition	(Mostly) Accurate	[25], [48]	Model size and nonlinear scaling behavior of models with coupled time steps: Reduce the number of time steps in one optimization run by splitting the time horizon into server sub-time-horizons.
Matrix decomposition	Accurate	[10]	Non-parallelizability of optimization problems: Decompose the optimization problem's matrix into independent blocks

Table 1: Non-exhaustive overview of techniques for reducing the complexity in ESOMs with a classifications of the resulting change compared to the original formulation, selected references that apply these techniques, and a description of the targeted complexity problem and the reduction approach.

Which components to include and which to omit is decided at the beginning of every modeling process by defining the system boundaries. Within the system boundaries, relations may be omitted or significantly simplified. Han et al. [34] omit the load dependency of coefficients of performance (COPs) of several conversion processes. Sarid and Tzur [35] assume constant losses per distance for AC lines and thereby omit their nonlinear dependency on the current and the amount of transmitted energy.

Approximation techniques can be used to reduce the depicted and the computational complexity while remaining part of the complexity of the original formulation [29]. Different approximation techniques can be applied to the same model formulation and are therefore subject not only to a trade-off between complexity and accuracy regarding the original formulation, but also regarding other approximation techniques. Linearization techniques are used to linearly approximate nonlinear relations. Lin et al. [22] compare different step-wise linearization techniques. Milan et al. [23] linearize partial load efficiencies using binary steps and SOS-constraints[4] and afterwards compare both approaches to constant efficiencies. Most optimization models discretize the time infinite dimension into time-steps with a certain step-length [50]. The number of time-steps may still cause complexity problems and is therefore further reduced by aggregation methods. Pfenninger [37] and Kotzur et al [24] aggregate time series data with clustering techniques, among other operations, to find a good trade-off between the retained accuracy and the reduced solving times. To allow time series aggregation for systems containing storage systems, Kotzur et al. [38] expanded this method to coupled time steps. Teichgraeber and Brandt [39] further formalize the choice of time series clustering methods and use the error in OFV as evaluation criteria. To reduce the amount of variables and constraints and to allow an efficient integration of dispatch into expansion planning models, Palmentier and Webster [9], [40], [41] cluster individual generation units to heterogeneous groups. The resulting error in OFV is small compared to the gain in reduced solving times of up to 2000%. Morales-Espana and Tejada-Arango [42] introduce a unit clustering technique that more realistically restricts the individual units' operational flexibility in the clustered groups.

Reformulation techniques, such as the Big-M method[5] or adding artificial cutting planes [52], represent the original relation in a different way, usually by adding additional discrete variables and constraints. Therefore, the usage of reformulation techniques is again subject to a trade-off between their advantage in, e.g., linearizing a relation and increasing the model size [52]. The Big-M method is applied in most MILP models, e.g. to implement an if-then-else constraint to limit the possible generation of a unit if it was selected from a range of possible options [43] or to implement an either-or constraint for restricting the operation to an interval between minimum and maximum power [36]. Yang et al. [44] compare different, equally accurate formulations for modelling the start-up and shut-down status of thermal generation units by the

[4] A special ordered set (SOSs) is an efficient implementation for a piecewise linear approximation predefined by most solvers [49]

[5] For an exhaustive introduction of the Big-M method in integer programming see [51]

resulting solving times. Decomposition methods divide the normally non-parallelizable optimization problem into sub-problems. A decomposition is possible along interfaces that are either derived from correlations of the real system or along interfaces that are offered by the structure of the mathematical problem. In the first case, an investment problem can be solved first, and the solution to this problem then represents the constraints for the dispatch problem [47]. In addition, more sophisticated methods, such as Benders decomposition[6] can be applied to decompose an optimization problem into smaller sub-problems [45], [46]. To reduce the number of time steps and approach the issue of perfect foresight in ESOMs, the time horizon can be split into sub-time-horizons by applying rolling horizon [25] or myopic foresight [48]. For the second case, the structure of the matrix is searched for independently solvable partial blocks, which can then be solved in parallel on high-performance computers [10].

2.2 Quantifying computational complexity and accuracy

Since there is no universal size for computational complexity in ESOMs, a range of indicators is used in literature for its quantification. These can be divided into indicators for computational time and space complexity [54]. Time and space complexity are analyzed in theoretical computer science by distinguishing between worst-case, best-case, and average time or space complexity [55]. While worst-case, best-case, and average time or space complexity (worst-case complexity is in many cases used synonymously for computational complexity and is depicted with the Landau symbol $O(\cdot)$, see [56]) theoretically analyze the relation between the model size and the (run) time or (memory) space required to solve a problem on it, they are not based on empirical experiments. An empirical analysis would be much more practical [57] and is applied in this paper.

The way an empirical time complexity is quantified is again a matter of choice. One way would be to measure computational complexity as the running time for simulating or solving a mathematical problem [26]. Since the running time depends on the specific computer system used, it is hardware-dependent. Another option is to measure the number of operations (e.g. floating point operations – FLOPS), that can be regarded as hardware independent with each operation or iteration requiring a fixed amount of time, depending on the computer system [57]. The space complexity, i.e. memory space required during the computation, can be measured directly (empirically) or by using a proxy, e.g. the model size. For both time and space complexity, quantifying the computational complexity empirically is based on proxies for the complexity of the operations performed during computation [15].

Quantifying the accuracy of a model strongly depends on the objective of the analysis performed [26]. In general, accuracy is the result of a validation [58]. It can be quantified as the deviation from the results of a benchmark model (as applied in e.g. [25], [39]) or as the deviation from historical or experimental data on the behavior of the real-world system (as applied in e.g. [57]). For measuring the accuracy of time series different error measures can be applied [59]. The choice of the accuracy indicator is a decision made by the modeler and should, in our opinion, be related to the research objective.

3 Methodology: Description of the evaluation framework and the case study

The suggested framework for the evaluation of the trade-off between accuracy and complexity in ESOMs is depicted in Figure 2. The initial step is the generation of alternative model formulations. For this, model components must be identified that can be varied in their implementation. Different options for implementing the components are then collected. By combining all possible options, the number of alternative models results from a multiplication of the options per component. Since this number can get very large, an expert selection of a

[6] Benders decomposition is an iterative decomposition method named after Jacques F. Benders [53]

range of representative models is suggested. After optimizing the selected models, the respective accuracy and complexity indicators are extracted. Models that are Pareto optimal regarding these indicators are then identified[7]. In the following sections, a case study of PSOMs is presented that implements this framework.

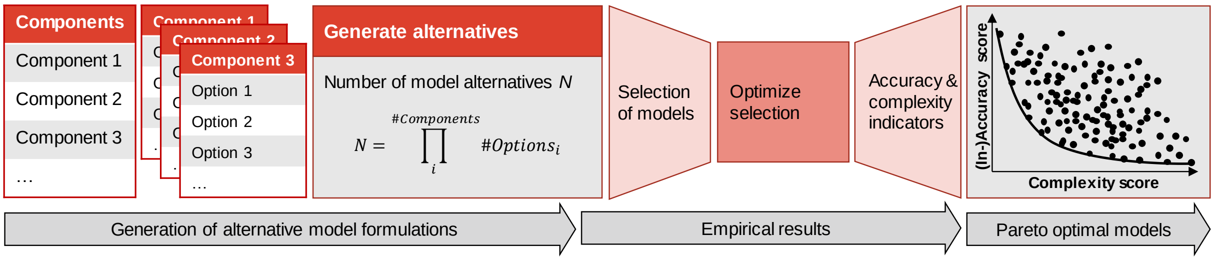

Figure 2: Framework for evaluating the trade-off between complexity and accuracy in energy system optimization model – the full process is divided in (1) the generation of alternative model formulations, (2) the optimization of selected model formulations, and (3) the evaluation of the complexity and accuracy indicators with the Pareto frontier

3.1 The developed modular and scalable power system optimization model

To identify complexity and accuracy drivers and to analyze the relationship between complexity and accuracy in PSOMs, a broad data basis covering different implementations, with different spatial and temporal resolutions, is required. For this purpose, a PSOM for an electricity distribution system is developed. It has a modular design in its component property implementations and an automatized preprocessing to apply the model at different temporal and spatial resolutions. In terms of power supply systems, a range of components and their properties are identified that are common in most PSOMs. The implementations of these components and properties come with complexity problems such as nonlinearities, binary variables, dynamics or large model sizes. By applying methods for complexity reduction, alternative implementations are found and the complexity problems inherited in the original formulation is addressed. This matching is depicted in Table 2 together with references that apply the respective complexity reduction methods. The mathematical formulations for the different implementations are provided in the supplementary material.

Property	Property	Complexity problem	Implementations
Conversion units	Partial load efficiency	Nonlinearities	Nonlinear
			Linearized with constant loss factors [61]
			Piecewise linearization with interpolation [22], [23]
			Piecewise linearization without interpolation [23]
			Constant [34]
	Minimum load	Binary variables, nonlinearities	Implemented [62]
			Not implemented
	Start-up, shut-down	Binary variables, dynamics	Implemented [62]
			Not implemented
	Minimum down times	Binary variables, Dynamics	Implemented [62]
			Not implemented
	Ramping rates	Dynamics	Implemented [62]
			Not implemented
Storage units	Storage level	Dynamics	Perfect foresight
			Dynamic [62]
Grids	Grid model	Model size, Nonlinearities	Copper plate assumption
			Transshipment grid model [35]
	Spatial resolution	Model size	Scalable [40], [63]
		Model size	Implemented [40]

[7] A solution is Pareto optimal if there is no other solution that would improve all the observed measures [60]

	Plant clustering		Not implemented
Time steps	Time horizon	Model size, influence on dynamics	Adjustable [24], [37], [39]
	Temporal resolution	Model size, influence on dynamics	Scalable [37]
Decision formulation	Objective function	Nonlinearities, dynamics	Dispatch planning
			Investment planning

Table 2: Different implementations for selected component properties in power system optimization models. For each property the associated complexity problems in power system optimization models are listed.

The technical components are classified into components for conversion units, for storage units, and for the electricity grid. Each of the components' properties has different implementation variants. The temporal resolution is varied by aggregating time-step information within a given time interval using averaging as shown in Figure 3. The spatial resolution is varied using (1) predefined spatial clusters as shown in Figure 4 and (2) k-means clustering methods applied to the individual conversion and storage units at each node, with four clusters per conversion technology [64]. We apply unit clustering in all our model variants due to the major potential for complexity reduction and the low negative impact on the accuracy we observed. Further, the spatial resolution defines the level of detail regarding the electricity transmission grid: In higher aggregation levels, transmission lines are clustered, reducing the number of nodes under consideration.

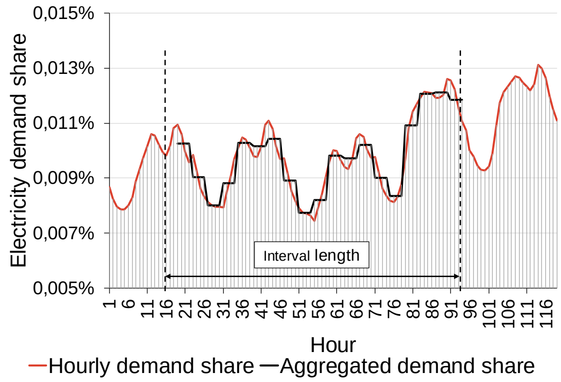

Figure 3: Temporal aggregation of a time series using averaging methods

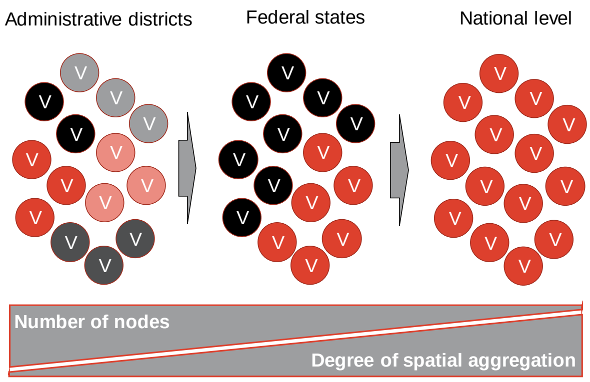

Figure 4: Spatial aggregation of nodes by using predefined clusters

We validate our model by using data from public sources on the German electricity supply system in 2016 and compare the historical electricity prices and dispatch schedules to the results generated by our model (so-called back casting). Times series data is derived from ENTSO-E [65], energy demand distribution among nodes from *Federal Working Group on Energy Balances* (*LAK*) [66] and *Federal and State Statistical Offices (StABL)* [67], transmission grid vertices and lines from SciGRID [68], geocoding information from [69], and data on conversion and storage units from [70], [71]. The model parameters that are required for the implementation of operational constraints can be found in literature on exergo-economics. For example, Mondal and Ghosh analyze a combined cycle biomass plant unit and calculate and provide different economic and technologic parameters [72]. Khanmohammadi et al. analyze a steam power plant and calculate costs related to exergy-destruction for different plant components [73]. The electricity prices are extracted from the ESOM using dual solutions of the supply-demand balancing constraint [74]. The dual solutions at all observed nodes are compared and aggregated according to the merit-order-based dispatch and zonal pricing used in Germany [75]. As shown in Figure 5, the electricity prices calculated by the model come close to the historical electricity prices with a mean absolute error[8] of 3.03 €/MWh

[8] Defined as $\frac{1}{n}\sum_{i=1}^{n}\left|p_i^{historic} - p_i^{model}\right|$ where n is the number of time steps and p the price

and an mean absolute percentage error of the mean price[9] of 10.47%. Currently, the model fails to simulate unusually high and low electricity prices. However, for the purpose of this paper, the accuracy in representing historical market decisions deems sufficient.

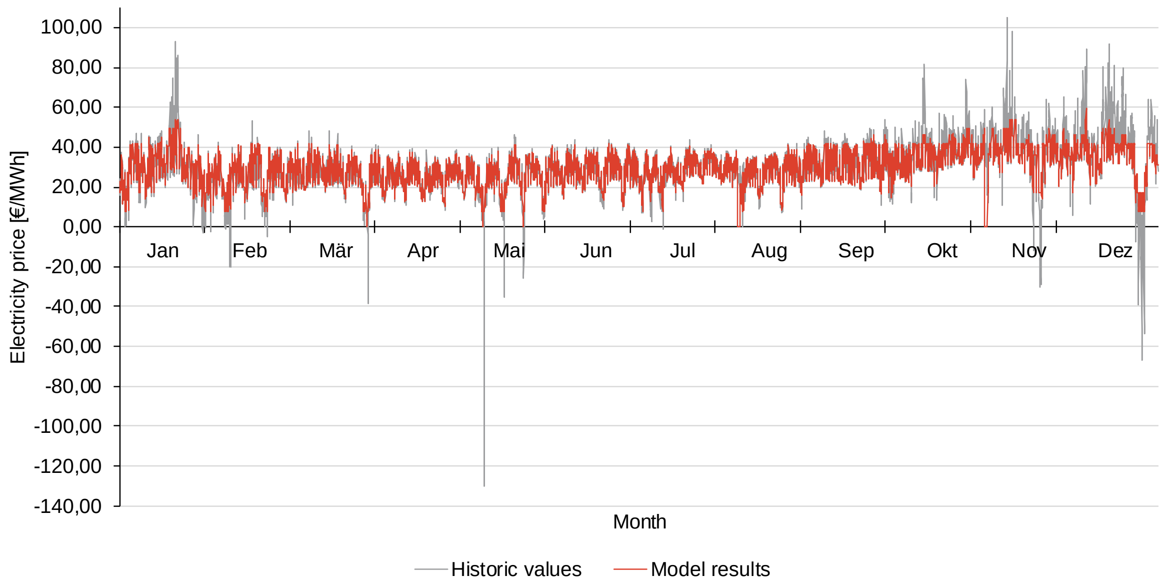

Figure 5: Model results for electricity prices in Germany in 2016 compared to observed historic electricity prices during this year

3.2 Selection of model settings and evaluation criteria

All optimization runs are performed on the same computer with an i5-5287U CPU 2.9 GHz and 8 GB Ram memory storage. The models are formulated in the Python programming language and optimized using the SCIP solver [76]. All problems are solved to optimality with a solving time limit set to 7,200 s. The model currently allows 11,520 combinations of technical settings if three different spatial and temporal resolutions are considered. This range is reduced by defining twenty representative technical settings and several temporal and spatial resolution settings as summarized in Table 3 and Table 4. The modular settings are grouped into models analyzing individual technical component properties by keeping other properties at a basic level and a selection of use cases (UCs) that analyze properties collectively. Hereby we include different complexity relevant aspects in a limited number of models – these are the behavior of individual component properties, the behavior of detailed operational constraints, the impact of different grid models, the behavior of storage constraints in models with varying complexity, and finally the model behavior at different temporal and spatial resolutions. Separating settings that implement single complex component properties and use cases that combine several complex component properties allows to separate the influence of the individual implementation from the behavior that occurs only when certain implementations are combined. The nonlinear implementation for partial load efficiencies is excluded since it was not shown to be practicable in terms of solving times. The dispatch models are optimized for an interval of two days and the investment models for an interval of three years.

Modular setting	**Coefficient of performance(COP)**	**Minimum Load (ML)**	**Start-up/ shut-down (ST)**	**Minimum down-time (MD)**	**Ramping rates (RA)**	**Storage level (DYN)**	**Grid model**
COP-C	Constant	Inactive	Inactive	Inactive	Inactive	Foresight	Copper
COP-CL-ML	Con. loss	Active	Inactive	Inactive	Inactive	Foresight	Copper
COP-B	Binary	Active	Inactive	Inactive	Inactive	Foresight	Copper
COP-BI	Bin. Inter.	Active	Inactive	Inactive	Inactive	Foresight	Copper

[9] Defined as $\frac{1}{\langle p^{historic}\rangle}\frac{1}{n}\sum_{i=1}^{n}\left|p_i^{historic}-p_i^{model}\right|$ where *n* is the number of time steps and *p* the price

ML	Constant	Active	Inactive	Inactive	Inactive	Foresight	Copper
ST-MD	Constant	Inactive	Active	Active	Inactive	Foresight	Copper
ST-RA	Constant	Inactive	Active	Inactive	Active	Foresight	Copper
DYN	Constant	Inactive	Inactive	Inactive	Inactive	Dynamic	Copper
TRANS	Constant	Inactive	Inactive	Inactive	Inactive	Foresight	Trans.
UC-C-1	Constant	Active	Active	Active	Inactive	Foresight	Copper
UC-C-L-1	Constant	Active	Active	Active	Inactive	Foresight	Trans.
UC-C-2	Constant	Inactive	Active	Active	Active	Foresight	Copper
UC-C-L-2	Constant	Inactive	Active	Active	Active	Foresight	Trans.
UC-C-3	Constant	Active	Active	Active	Active	Foresight	Copper
UC-C-4	Con. loss	Active	Active	Active	Active	Foresight	Copper
UC-C-L-3	Constant	Active	Active	Active	Active	Foresight	Trans.
UC-C-L-4	Con. loss	Active	Active	Active	Active	Foresight	Trans.
UC-D-1	Constant	Inactive	Inactive	Inactive	Inactive	Dynamic	Trans.
UC-D-2	Con. loss	Active	Inactive	Inactive	Inactive	Dynamic	Trans.
UC-D-3	Constant	Active	Active	Active	Active	Dynamic	Copper

Table 3: Selected technical settings out of 1,280 possible technical alternative combinations.

Resolution setting	Grid resolution	Time resolution (dispatch/invest)	Unit clustering
1	National	4 hours / 28 days	Active
2	National	2 hours / -	Active
3	National	1 hour / -	Active
4	Federal	4 hours / 28 days	Active
5	Federal	2 hours / -	Active
6	Federal	1 hour / -	Active
7	Administrative district	4 hours / 28 days	Active

Table 4: Selected temporal and spatial resolutions that are used in combination with the selected technical settings.

After optimizing the previously defined models, indicators for complexity and accuracy are extracted from the optimization statistics and results. The indicators used in this paper are listed in Table 5. The first group of indicators is used to quantify the complexity of the model variant. The complexity indicators are grouped into hardware-dependent indicators (absolute solving time) and hardware-independent indicators (memory usage, model size, variable number and types, and constraint number and types). The model size is defined as the model's number of tableau entries [77]. The accuracy indicator used is the deviation in OFV from a benchmark model variant (UC-C-L-4 with a temporal resolution of 3600 s and spatial resolution of federal states).

Indicator	Measuring unit	Hardware dependency
Solving time	Seconds	Yes
Memory usage	Megabyte	No
Model size	Number of variables and constraints	No
Deviation in objective function value	%	No

Table 5: Selected complexity and accuracy indicators for evaluating of the trade-off between complexity and accuracy among the selected power system optimization model formulations

4 Results

Having described the model setup, we present and compare the results as follows. First, we demonstrate results based on dispatch models. After presenting a full overview of the dispatch model results, the results are further separated into different system components and relations. Subsequently, we compare these results to those we obtain when investment planning is considered, to investigate the influence of the decision formulation on the model's behavior in terms of complexity and accuracy.

The relationship between solving time and deviation in OFV among the model variants that were successfully optimized is shown in Figure 6. The right-hand figure contains an excerpt of the solution range including settings with a solving time below 200 s. It contains 85% of all data points and 91% of the nondominated solutions. Most results scatter between 0 and 500 s solving time, while the accuracy varies greatly. To filter efficient and less efficient solutions, the Pareto optimal solutions are calculated. The data points are therefore divided into dominated solutions (i.e. not Pareto optimal solutions) and nondominated solutions (i.e. Pareto optimal solutions). The set of all Pareto optimal solutions, the Pareto frontier, is depicted in red. It can be observed that very accurate settings with a deviation in OFV of down to 0.3% require roughly 34 seconds and that above this duration, further increase in accuracy comes with very high solving times above 2,000 seconds. The majority of the Pareto optimal settings that are listed in Table 6, belong to the use case settings that implement a high degree of detail regarding operational constraints for conversion units. Also, some settings implementing dynamic storage constraints are listed. The dynamic storage implementation applied in this work restricts the flexibility of storage device which has an increasing influence on the OFV. This results in an overestimation of the total system costs compared to the benchmark implementations using perfect foresight for the storage unit dispatch.

Figure 7 illustrates the relationship between model size and deviation in OFV among the model variants that were successfully optimized. The right-hand figure depicts all variants with a model size of below 1 billion tableau entries which is 0.3% of the maximum tableau size and which comprises 47% of all settings tested and 90% of the nondominated solutions. The total model size range varies from 6.55 million to 326 billion tableau entries. The required memory usage is approximately linear with the model size (see Appendix A). Therefore, the model size can be interpreted as a proxy for the memory usage. Four out of ten Pareto optimal settings are use cases with high degree of detail in their operational behavior. Half of the Pareto optimal settings implement dynamic storage constraints. As we noted earlier, this can be attributed in part to increased system costs through reduced flexibility compared to settings without dynamic storage. A full table of the quantitative results obtained during this work can be found in Appendix B.

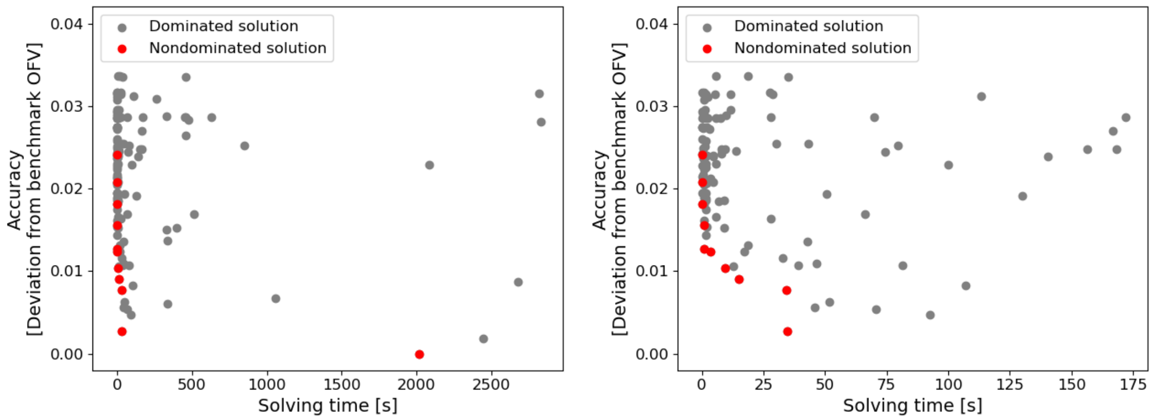

Figure 6: Optimization results for combinations of the complexity measure solving time and the accuracy measure deviation from the benchmark objective function value (OFV). The data points belonging to the Pareto front (i.e. dominant solutions) are highlighted. The right sub-figure shows all solutions with a solving time < 200 s.

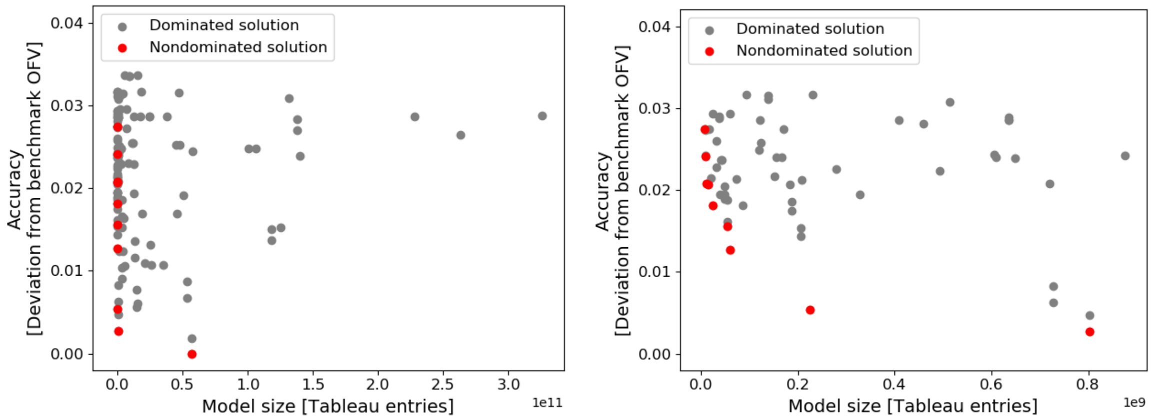

Figure 7: Optimization results for combinations of the complexity measure model size and the accuracy measure deviation from the benchmark objective function value (OFV). The data points belonging to the Pareto front (i.e. dominant solutions) are highlighted. The right sub-figure shows all solutions with a model size < 1e9 tableau entries.

Setting	**DYN**	**UC-D-L-1**	**UC-C-L-1**	**UC-C-L-4**	**UC-D-L-3**	**UC-C-L-1**	**UC-C-L-3**	**UC-C-L-4**	**UC-C-L-3**	**UC-C-L-4**	**UC-C-L-4**
Temporal resolution [s]	14,400	14,400	14,400	14,400	14,400	14,400	14,400	14,400	7,200	3,600	3,600
Spatial resolution	large	large	large	large	large	medium	medium	medium	medium	large	medium
Solving time [s]	0.05	0.08	0.14	0.76	0.99	3.35	9.34	15.14	34.11	34.69	2,016.31
Deviation in OFV	0.024	0.021	0.018	0.016	0.013	0.012	0.010	0.009	0.008	0.003	0.000

Table 6: Nondominated model settings, i.e. Pareto-optimal solutions for the trade-off between solving time and deviation in objective function value (OFV)

Setting	**COP-C**	**DYN**	**UC-D-L-1**	**UC-D-L-2**	**UC-C-L-1**	**UC-C-L-4**	**UC-D-L-3**	**UC-D-L-3**	**UC-C-L-4**	**UC-C-L-4**
Temporal resolution [s]	14,400	14,400	14,400	14,400	14,400	14,400	14,400	7,200	3,600	3,600
Spatial resolution	large	large	large	large	large	large	large	large	large	medium
Model size [Tableau entries]	6.6E+06	8.4E+06	1.1E+07	1.4E+07	2.3E+07	5.4E+07	5.9E+07	2.3E+08	8.0E+08	5.7E+10
Deviation in OFV	0.0275	0.0241	0.0208	0.0206	0.0181	0.0156	0.0127	0.0054	0.0027	0.0000

Table 7: Nondominated model settings, i.e. Pareto-optimal solutions for the trade-off between model size and deviation in objective function value (OFV)

4.1 Part-load efficiency

Four implementations for part-load efficiencies are compared. Figure 8 shows the results regarding the solving time, the model size and the deviation in OFV for constant (COP-C), constant-loss (COP-CL-ML), binary (COP-B) and binary-interpolation (COP-BI) part-load efficiency implementations. The two binary implementations for partial load efficiency (COP-B, COP-BI) have the highest solving times and show infeasibility at least once within the predefined 7,200 s. The solving times of the COP-CL-ML model are to some extent influenced by the required minimum load implementation. The two binary implementations come with, on average, more than ten times the model size as the COP-C and COP-CL-ML models. Regarding accuracy, no major difference can be observed between the models.

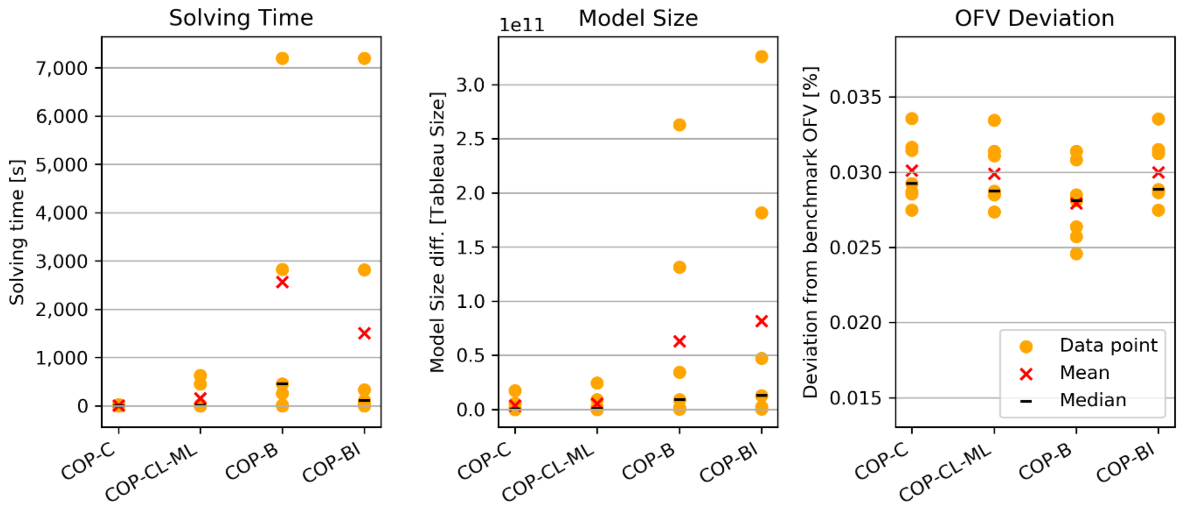

Figure 8: Distribution of the indicators solving time, model size and deviation in objective function value (OFV) for models varying the part-load efficiency implementation. The data points represent different temporal and spatial resolutions and are clustered by model settings.

4.2 Operational constraints

The degree of detail in modeling generation and storage units is varied by implementing minimum loads (ML), start-up costs and minimum down times (ST-MD), as well as ramping rates that are combined with start-up costs (ST-RA). Four different use cases (UC-C-1, UC-C-2, UC-C-3, and UC-C-4) combine several of the aforementioned implementations. The base model (COP-C) does not include any of the detailed implementations and is used as a benchmark. The results are depicted in Figure 9. The degree of detail in modeling generation and storage units has a high impact on solving time. Individually, minimum load implementations increase the solving time the most. In combination, however, the impact of minimum load implementations is less than the impact of combined dynamics. The use case UC-C-1, that implements minimum loads, start-up costs and minimum down-times, shows lower solving time than the use case UC-C-2. The latter implements ramping rates instead of minimum loads. The combination of dynamics, here in particular minimum down-times and ramping rates, seem to have a significant impact on the solving time.

Regarding the model size, ramping rates have the highest influence, followed by minimum down-times and minimum loads. Accuracy is highest for the very detailed models. The ramping constraints have an interesting impact on the accuracy in that they come with lower dispersion in OFV across different temporal and spatial resolutions.

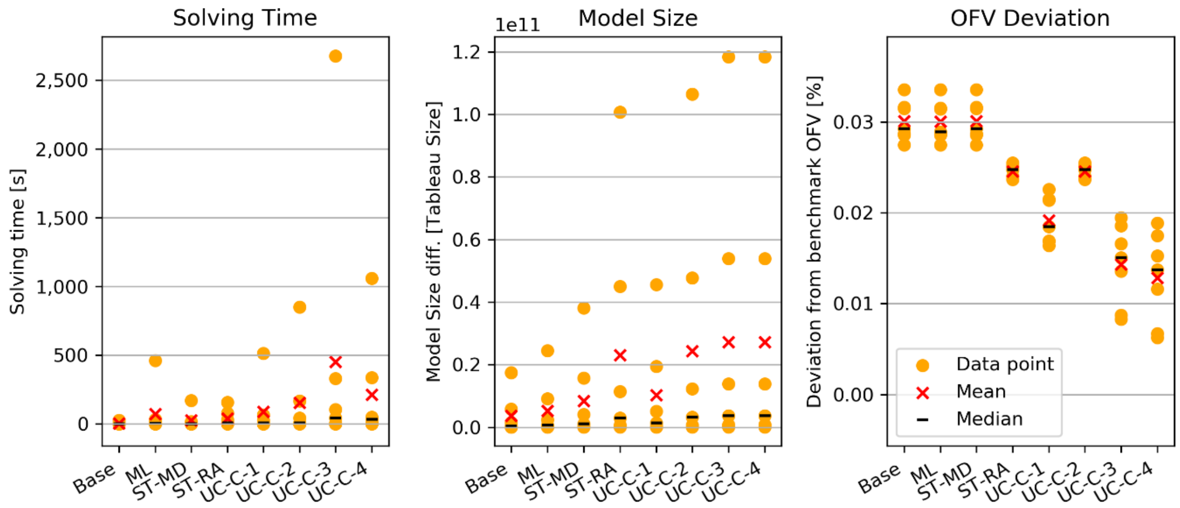

Figure 9: Distribution of the indicators solving time, model size and deviation in objective function value (OFV) for models varying the degree of detail in modeling generation and storage units. The data points represent different temporal and spatial resolutions and are clustered by model settings.

4.3 Transshipment grid model and copper plate

The transshipment grid model implementation, which assumes linear losses along the lines, is compared with a simple copper plate implementation while all other implementations are kept constant. The differences in complexity and accuracy indicators are presented in Figure 10 . This compares the results of the transshipment grid model with the results of the copper plate implementation. The differences are calculated by comparing the indicators against each other, such that the difference is $(Ind_{TG} - Ind_{CP})/Ind_{CP}$, where *Ind* stands for an indicator, *TG* for the transshipment grid model and *CP* for the copper plate model. A positive difference states that the respective indicator is higher for the transshipment grid model than for the copper plate model. The difference in solving times varies from -86% to +135%, while the average difference ranges from -30% to +30%. The solving times for more complex models (i.e. UC-C-(L)-2, -3, and -4) show on average higher solving times with the transshipment grid model than with the copper plate model. The model size is increased by the transshipment grid model. However, the effect of this influence decreases with the overall model size. The accuracy is lower for the transshipment grid model than for the copper plate model, though the average difference is low at around 0.05 %.

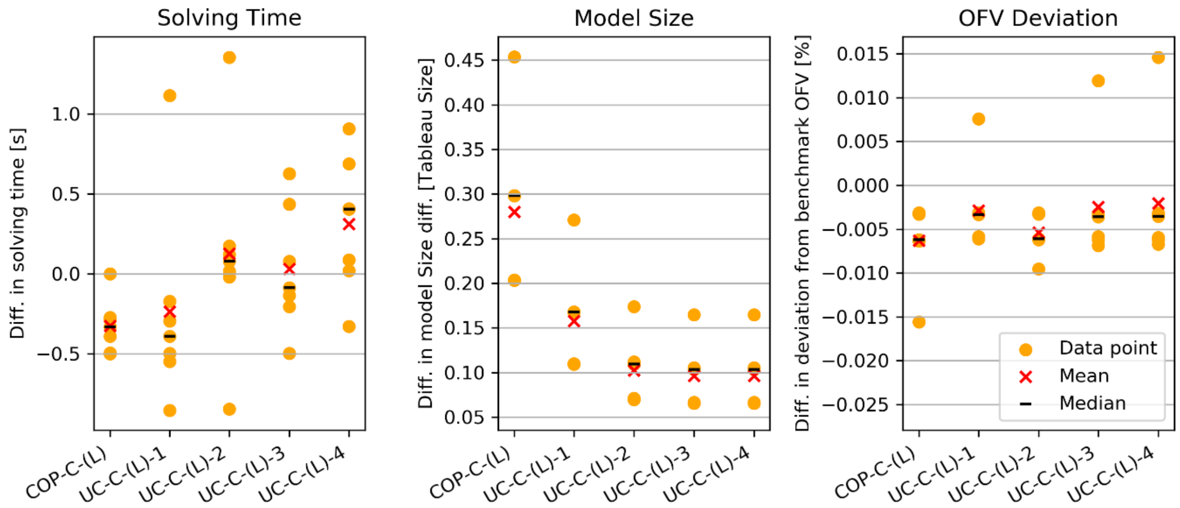

Figure 10: Comparison of the indicators solving time, model size and deviation in objective function value (OFV) for models implementing the transshipment grid model to models using a copper plate model. The data points represent different temporal and spatial resolutions and are clustered by model settings.

4.4 Dynamic storage level

The dynamic storage variables pass a storage level between the time steps (i.e. they are recursively dependent on previous time-steps). This implementation is checked against respective models that use perfect foresight for the storage unit dispatch and that do not require the dynamic storage level variable. Again a positive difference in the graphs in Figure 11 implies that the respective indicator is higher for the models using the dynamic storage level than for the models using perfect foresight on storage dispatch. The results for the solving times show that without any other dynamic implementations present, the dynamic storage unit implementation has only a small influence. However, it should be taken into account that the dispatch models were only optimized for a maximum of 48 time steps. The influence should increase with the number of time steps. When the dynamic storage implementation is combined with other dynamic implementation as done in the use case UC-C-D-3, the results show that the solving times increase on average by a factor of 16. The model size is higher for models using the dynamic storage level implementation, but this influence decreases with the overall model size. In general, the impact on accuracy is low for the basic models and varying among different spatial and temporal resolutions for the detailed use case UC-D-3. The dynamic storage level implementation adds further limitation on the flexibility of storage units which increases the total system costs. This can lead to unrepresentative results for models with this implementation if the deviation in OFV from a benchmark model without this implementation is used as an accuracy indicator. Therefore, besides a valid complexity assessment, the accuracy evaluation for dynamic storage level implementation using the applied indicator can be misleading.

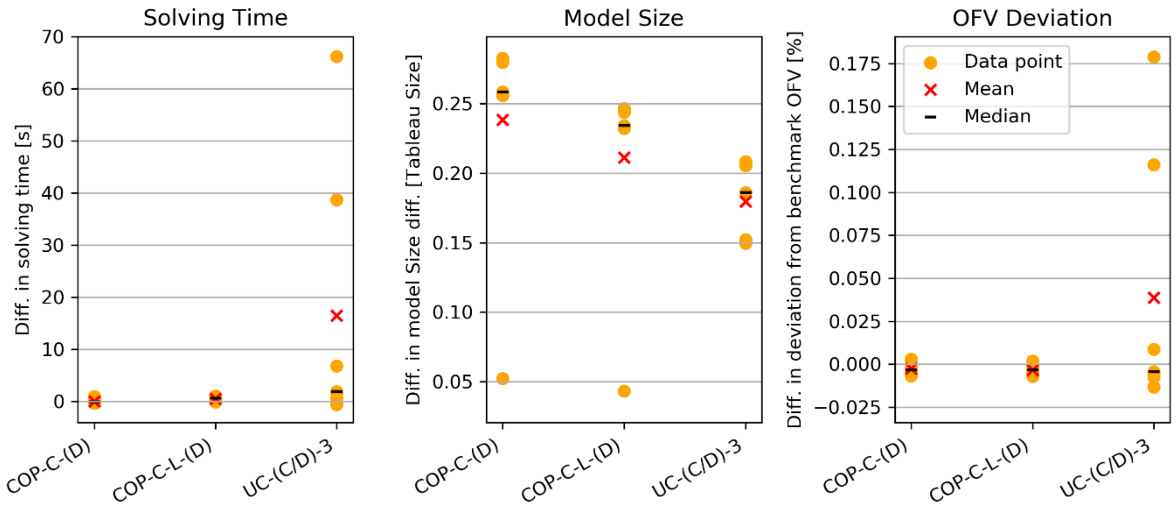

Figure 11: Comparison of the indicators solving time, model size and deviation in objective function value (OFV) for models implementing a dynamic storage level to models assuming perfect foresight for storage dispatch. The data points represent different temporal and spatial resolutions and are clustered by model settings.

4.5 Temporal aggregation

The results shown in Figure 12 vary in their temporal resolution by carrying the length of a single time step. All models have a fixed spatial resolution on the level of federal states. Though in all temporal resolutions the time limit of 7200 s for the optimization was reached, it can be observed that the solving time depends strongly on the temporal resolution. While the solving time seems to scale nonlinearly with the temporal resolution, the model size increases quite linearly. The temporal resolution increases the accuracy in terms of deviation in OFV except for one outlier.

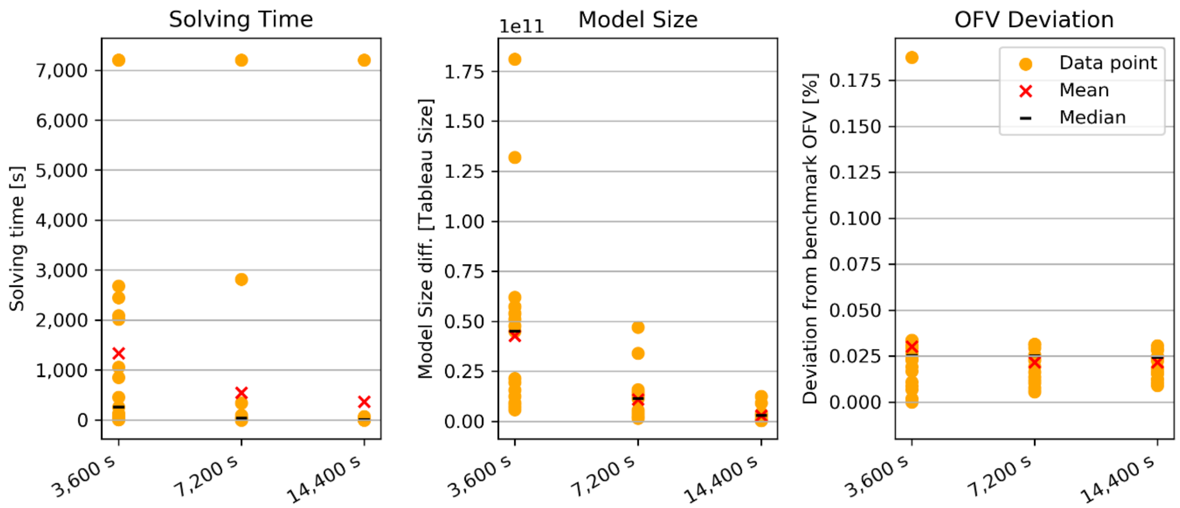

Figure 12: Comparison of the indicators solving time, model size and deviation in objective function value (OFV) for temporal resolutions of 3600 s, 7200 s, and 14400 s per time step with a fixed spatial resolution on federal state level. The data points represent different model settings and are clustered by temporal resolution.

4.6 Spatial clustering and aggregation for transshipment grid model

Figure 13 depicts the results for varying spatial resolutions with a fixed temporal resolution of 14,400 s per time step. Only the models using the transshipment grid model are compared in this section due to their stronger dependency on the spatial resolution. The number of nodes on the level of administrative districts is 394, on the level of federal states it is 16 and there is a single node on the national level. The solving time roughly correlates with the number of nodes – the models on the level of federal states require on average 15 times the solving time and the models on the level of administrative districts 333 times the solving time compared to the models on the national level. In contrast, the model size is 69 times higher for models on the level of federal states and 2,520 time higher for models on the level of administrative districts compared to models on the national level. The deviation in OFV does not change significantly among the spatial resolutions.

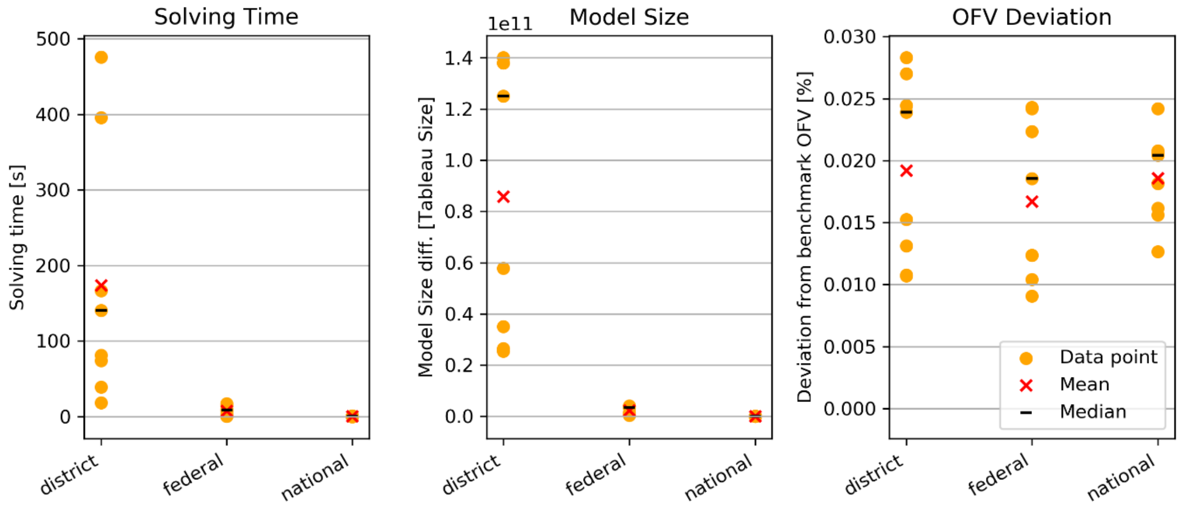

Figure 13: Comparison of the indicators solving time, model size and deviation in objective function value (OFV) for spatial resolutions on the level of administrative districts, federal states and country with a fixed temporal resolution of 14,400 s per time step. The data points represent different model settings and are clustered by spatial resolution.

4.7 Comparison with models for expansion planning

Having focused on dispatch models, we now consider investment decisions. Table 8 shows the results for the technical settings using 28-day time steps and a spatial resolution of national level. The minimum OFV among all feasible models lies 0.05% below the maximum. Hence, all feasible settings tested calculate similar system costs. The technical setting influences the number of variables and constraints. While the number of integer variables is equal for all settings (it is influenced only by the spatial resolution), the number of binary variables strongly depends on the technical setting. The minimum load constraint increases the number of binary variables in this example by a factor of 29, the start-up costs by a factor of 85, the binary implementation for partial load efficiencies by a factor of 161, and the binary interpolation implementation for partial load efficiencies by a factor of 129. The number of constraints is influenced the most by the two binary implementations for partial load efficiencies and the ramping implementation.

In contrast to dispatch models, investment models implementing partial load efficiencies using constant losses cannot be solved within the defined timeframe. Instead, the respective implementation applying binary interpolation that performs badly for the dispatch models now shows comparably short solving times. The transshipment grid model does not significantly influence the solving time, as already observed for the dispatch models. Including minimum loads seems to be also a major factor influencing the solving time of invest models, followed by the ramping implementation. The dynamic storage implementations show good performance for the tested resolutions. However, testing them on longer intervals should again result in a large increase in solving time, though this was not tested in the context of this paper.

Settings	**Solving time [s]**	**Objective function value [bn €]**	**Number of Variables**			**Number of constraints**
			Con.	Bin.	Int.	
COP-C	2.71	1.4724E+10	1,596	14	96	2,177
COP-CL-ML	> 7,200	-	2,044	406	96	3,745

COP-B	> 7,200	-	3,836	2,254	96	8,001
COP-BI	16.13	1.4721E+10	5,628	1,806	96	7,553
ML	17.42	1.4729E+10	1,596	406	96	2,961
ST-MD	4.17	1.4724E+10	1,596	1,190	96	2,569
ST-RA	10.39	1.4724E+10	1,988	1,190	96	4,909
DYN	2.87	1.4724E+10	1,764	14	96	2,439
TRANS	2.39	1.4724E+10	1,848	14	96	2,429
UC-C-1	45.69	1.4729E+10	1,596	1,190	96	3,353
UC-C-L-1	27.77	1.4729E+10	1,848	1,190	96	3,605
UC-C-2	9.03	1.4724E+10	1,988	1,190	96	4,909
UC-C-L-2	9.71	1.4724E+10	2,240	1,190	96	5,161
UC-C-3	41.5	1.4729E+10	1,988	1,190	96	5,693
UC-C-4	> 7,200	-	2,436	1,190	96	6,477
UC-C-L-3	58.06	1.4729E+10	2,240	1,190	96	5,945
UC-C-L-4	> 7,200	-	2,688	1,190	96	6,729
UC-D-L-1	4.97	1.4724E+10	2,016	14	96	2,691
UC-D-L-2	> 7,200	-	2,464	406	96	4,259
UC-D-L-3	94.58	1.4729E+10	2,408	1,190	96	6,207

Table 8: Optimization results for different invest models with different modular settings – optimized for one year using 28 days long time steps and node resolution on a national level

5 Discussion

Based on our results, we evaluate the individual components and their parameter implementations to answer the research questions we posed. In what follows, the results for the proxy model size are again referred to as memory usage. The propositions made must be interpreted in the context of the model and scenario analyzed in this paper and might not be entirely transferable to different problems and methodologies. Nonetheless, our systematic approach towards the assessment of the relationship between complexity and accuracy in PSOMs is unique.

5.1 Are complex power system models more accurate?

The shapes of the Pareto frontiers indicate that yes, the accuracy of the Pareto optimal solutions increases with complexity. Regarding the complexity indicator solving time, 91% of the Pareto optimal solutions require less than 35 seconds – which is 1.2% of the maximum solving time observed among the successfully solved models. A similar observation can be made for the complexity indicator model size as a proxy for memory usage. 90% of all Pareto optimal solutions have a model size of less than 900 million tableau entries – that is 0.3% of the maximum model size. The distinction between Pareto optimal and dominated solutions shows that most of the possible model formulations should be neglected in favor of those with a more balanced trade-off between complexity and accuracy. The marginal utility in the form of higher accuracy decreases with additional complexity. The findings lead to our conclusion that there is a tendency for a high degree of accuracy requiring a certain degree of complexity – but models with relatively low complexity can already provide sufficient accuracy. Among the Pareto optimal solutions, the majority of models contain a high degree of detail in the operational constraints of conversion units. The models with dynamic storage constraints, i.e. time-coupled storage levels, being among the Pareto optimal solutions can be explained by the increase in total system costs caused by the additional limitation of the flexibility of storage units. This additional limitation is not implemented in the benchmark setting. This shows, the dependency of the accuracy measurement on the selected benchmark setting. However, the benchmark settings selected is the most accurate one since it depicts the actual storage unit behavior by using perfect foresight. Overall, the implementation of a high level of operational detail should be preferred over a high level of temporal and spatial detail in PSOM for dispatch planning. From the small difference in OFV in the investment models analyzed it can be concluded that detailed operational behavior of conversion units do not significantly improve the accuracy of the results.

5.2 What are the complexity and accuracy drivers in ESOMs?

For the given conceptual model and scenario, the choice of implementation for partial load efficiency contributes little to a change in the accuracy indicators. However, in terms of the observed complexity indicators, i.e. solving time and memory usage, the impact of the implementation for partial load efficiency is high. Binary efficiency implementations add a large number of binary variables to the model which leads to a major increase on solving time and memory usage. This effect increases for higher spatial resolutions that come with a higher number of conversion and storage units. The best alternative implementation for constant efficiencies are using constant loss factors in dispatch models and using binary interpolation in investment models.

Among the detailed conversion and storage unit implementations, minimum loads have the largest influence on the solving time, while including ramping has the largest influence on the memory usage. In terms of accuracy, the implementations for minimum loads, start-up costs and minimum down-times do not significantly influence the observed indicators if tested individually. Only the ramping implementation has a significant effect on the deviation in OFV. Additionally, the ramping implementation decreases the dispersion in OFV among different temporal and spatial resolutions. Ramping constraints should, therefore, be included in PSOMs if possible.

Two storage unit implementations are compared: consideration of perfect foresight for dispatched power by storage units and dynamic storage level implementation. Tested individually, the dynamic storage implementation has low influence on solving time and memory usage. However, the number of time steps tested is comparably small and the scaling behavior indicates that for a larger number the solving time will be affected more severely.

The grid model implementations analyzed are the copper plate model and the transshipment grid model. The difference in solving times between the two implementations varies widely, however, the models that are more complex in the operational behavior of conversion units show on average higher solving times with the transshipment grid model than with the copper plate model. Positive effects of the transshipment grid model on the solving time might be due to an increase in tightness of the model (despite the loss in compactness) by adding additional constraints that restrict the exchange of electricity, thereby reducing the feasible area of the relaxed linear programming (LP) problem while solving the MILP problem [62]. The memory usage for settings using the transshipment grid model is slightly higher than for settings using the copper plate model, though this influence decreases when the overall model size is high. Including the transshipment grid model in PSOMs for electricity supply systems can, therefore, make the model more realistic and also contribute to reducing the model complexity for some model variants.

The joint analysis of different component property implementation allows evaluating how these influence one another in terms of complexity and accuracy. The analyzed settings using combinations of minimum loads, start-up costs, minimum down-times, and ramping rates were shown to have a significantly higher influence on accuracy compared to the model variants that implement only one of the properties. It seems to be necessary to either not use detailed implementations for conversion units (ramping constraints might be an exception, here) or use a combination of different detailed implementations.

The use cases showed that combining implementations that individually come with dynamics (i.e. time-dependent variables) increases solving time significantly. Implementing properties with dynamic behavior, such as start-up costs, ramping rates, or dynamic storage levels should be considered carefully and added piece by piece to avoid the negative effects of interfering dynamics.

The two observed decision formulations, (1) dispatch planning and (2) combined dispatch and investment planning, behave differently in three major aspects. First, the deviation among the OFV for invest models is lower than for pure dispatch models. This indicates that the implementation of technical components is less relevant in invest models. Secondly, the number of installed units becomes a decision variable, leading to nonlinear constraints. These are linearized by using, e.g., Big-M reformulation, which comes with an increase in model size and especially an increase in the number of discrete variables. However, this partly changes the performance of the model. This can be seen in the third aspect, the different behavior of partial load implementations. The constant loss implementation for partial loads performs much worse in invest models, while the binary interpolation implementation here shows to be the best alternative to constant efficiencies.

The decision formulation has a significant influence on the complexity and accuracy behavior of ESOMs, even if the same energy system is investigated. Thus, the model developer should take into account the specific formulation requirements that are accompanied by the decision formulation (and in a broader sense with the initial research question).

5.3 Limitations

As explained at the beginning of this chapter, all results depend on the conceptual model used, the specific implementation, and the scenario chosen. It is highly questionable that the propositions can be applied without changes to other ESOMs. The model used in this paper focusses on power supply systems and electricity markets in the context of Germany and Europe. The accuracy assessment is based on a single accuracy indicator, the deviation in OFV from a benchmark setting. Other accuracy indicators, such as dispatch schedules, electricity prices or electricity imports should be investigated in further studies. The results depend on the computer system they are tested on and on the solver used (currently SCIP, see [77]). Other systems and solvers may produce deviating results. However, keeping the system constant should allow propositions on the relative differences between model variants, which should be reproducible on other systems. The computer system used is limited in performance and therefore limited the model sizes that could be tested. The data basis used for the complexity and accuracy assessment, especially for invest models, is therefore limited.

6 Conclusion

In this paper, an approach was introduced that allows a systematic intra-model comparison of complexity and accuracy in ESOMs. The analysis provides beneficial information for (1) efficient use of computational resources and (2) a tailored design of power system optimization models for specific research questions. The results show that while more complex dispatch models tend to generate more accurate results, the marginal utility in the form of higher accuracy decreases with additional complexity. The complexity of most Pareto optimal solutions in terms of solving time and memory usage is relatively low. Therefore, the proposed framework allows to identify model implementations that should be omitted in favor of those with a more balanced trade-off between complexity and accuracy. Keeping in mind that all models tested in the intra-model comparison depict the same energy system and time interval, the variation in complexity is vast. Among all successfully tested dispatch models, the solving time ranges from 0.5 to 2,829 s and the memory usage ranges from 0.3 to 3,380 MB. Most of the Pareto optimal settings of dispatch models show temporal complexity (i.e. solving time) below 1.2% of the maximum temporal complexity and spatial complexity (i.e. memory usage) below 0.3% of the maximum spatial complexity. The total system costs in investment models – in contrast to economic dispatch models – hardly differ, indicating that complexity can be more easily reduced in investment than in economic dispatch models. The findings emphasizes the need for systematic evaluation of complexity energy system optimization models to reach the goal of parsimony.

Based on the representative models evaluated, a number of recommendations for power system optimization models could be derived. Including the transshipment grid model can in some cases make the model more realistic and contribute to reducing the model complexity in some settings by making the model formulation tighter. Among the different efficiency implementations, constant efficiencies are shown to be practicable for dispatch and especially invest models. If more realistic implementations are required, the efficiency implementation using constant losses performed well for dispatch models, while in invest models the efficiency implementation using binary interpolation (piece-wise linearization) is shown to be the best alternative for constant efficiencies. In general, the joint implementation of several detailed conversion unit implementations, such as minimum loads and ramping rates, contribute to accuracy in depicting generation and storage units. However, this comes with a high price in terms of an increase in complexity that is compensated in the Pareto optimal settings for dispatch models by lower degree of detail in temporal and spatial resolutions. Implementations adding dependencies between time steps, such as dynamic storage level or ramping implementations, can cause complexity problems, especially if several of these are combined with each other. Finally, the temporal resolution is shown to have a greater influence on the accuracy indicator compared to the spatial resolution. Scaling the spatial resolution causes a highly nonlinear increase in required memory space if the optimization model implements a grid model. The application of plant clustering methods leads to significantly reduced spatial complexity.

By structuring complexity research from the context of energy system analysis and by proposing a framework for systematically evaluating the trade-off between complexity and accuracy in energy system optimization models, we hope to have contributed to and motivated further research in this area. Our case study for choosing the power system optimization model with a balanced trade-off between accuracy and complexity can be transferred to other kinds of energy system optimization models. The shortcomings of our current approach offer opportunities for further research. From a meta level, an inter-model complexity comparison (cross-model comparison) that compares the use of optimization models to other conceptual models is of interest, especially with regard to the more frequent use of optimization models in energy system analysis [2]. Additionally, the intra-model complexity comparison should be extended to include more components, properties, and energy sectors as well as other methods for technological simplification, complexity reduction, and reformulation. Including other energy sectors may provide further opportunities for the use of complexity reduction techniques, such as decomposition approaches. Finally, the intra-model comparison should be performed using different solvers to analyze their influence on the results, as already started in [78].

Acknowledgment

The work has been carried out as part of the research project METIS which is funded by the German Federal Ministry for Economic Affairs and Energy under Project ID 03ET4064.

Appendix A

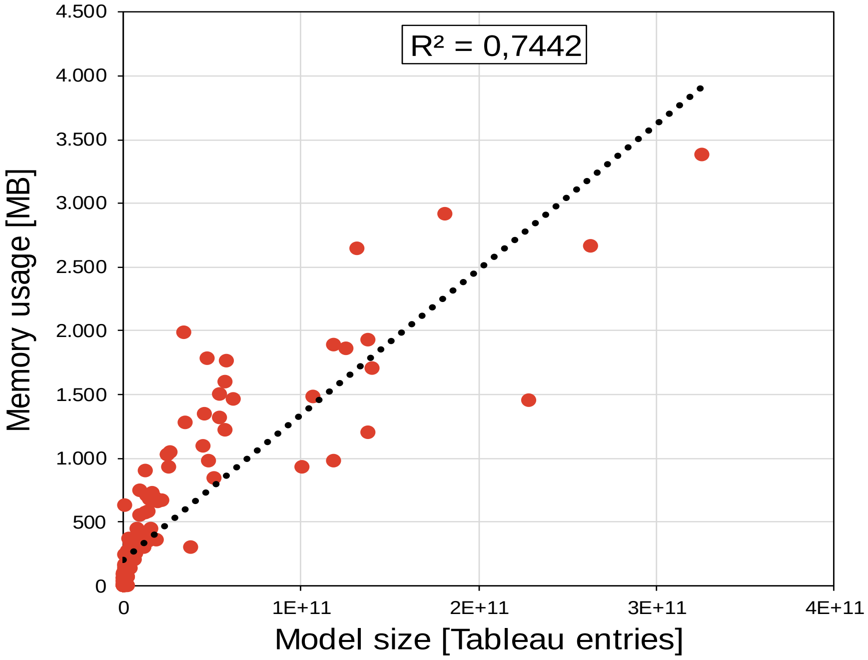

Figure 14: Relationship between the model size and the memory usage for the memory usage in the models optimized

Appendix B

Setting	Solving Time [s] 1	2	3	4	5	6	7
COP-C	0,07	0,22	0,29	1,38	1,74	5,83	27,98
COP-CL	0,38	1,36	2,32	7,51	28,69	457,33	631,19
COP-B	1,75	2829,20	13,92			261,55	457,82
COP-BI	2,03	9,66	113,28	69,86	2816,27		330,79
ML	0,22	0,51	0,55	5,83	11,80	35,01	460,30
ST-MD	0,26	0,39	0,80	2,50	5,40	18,55	172,08
ST-RA	0,65	1,75	4,67	7,76	30,06	79,62	156,47
DYN	0,05	0,25	0,33	1,03	1,19	11,62	27,52
TRANS	0,05	0,16	0,29	0,69	1,06	2,94	18,68
UC-C-1	0,23	0,44	1,79	6,66	28,04	66,17	513,81
UC-C-L-1	0,14	0,93	1,48	3,35	12,66	46,55	74,25
UC-C-2	0,75	1,91	4,53	9,25	43,05	850,24	168,14
UC-C-L-2	0,84	2,06	4,60	9,06	50,50	129,97	395,71
UC-C-3	0,62	1,78	107,08	5,75	43,00	2677,89	332,24
UC-C-4	0,70	1,58	51,64	8,98	32,71	1057,96	338,65
UC-C-L-3	0,89	1,92	92,67	9,34	34,11	2444,16	166,78
UC-C-L-4	0,76	1,61	34,69	15,14	45,93	2016,31	475,83
UC-D-1	0,08	0,19	0,42	0,63	1,79	5,74	39,16
UC-D-2	0,43	1,75	3,58	7,80	100,16	2085,41	81,55
UC-D-3	0,99	70,79		17,08	335,83		140,61

Setting	Model Size [Million tableau entries] 1	2	3	4	5	6	7
COP-C	7	24	94	409	1.521	5.859	17.480
COP-CL	10	36	139	635	2.360	9.095	24.536
COP-B	123	459	1.769	9.183	34.153	131.599	262.860
COP-BI	170	636	2.452	12.608	47.012	181.385	325.822
ML	10	36	139	635	2.360	9.095	24.536
ST-MD	16	60	230	1.096	4.072	15.681	38.138
ST-RA	41	156	610	2.982	11.474	44.999	100.676
DYN	8	31	120	514	1.912	7.373	18.392
TRANS	9	32	122	492	1.830	7.052	25.410
UC-C-1	20	73	280	1.355	5.034	19.388	45.534
UC-C-L-1	23	85	327	1.503	5.585	21.513	57.880
UC-C-2	43	166	648	3.193	12.209	47.733	106.485
UC-C-L-2	48	184	719	3.419	13.063	51.047	124.992
UC-C-3	49	187	728	3.613	13.792	53.876	118.320
UC-C-4	49	187	728	3.613	13.792	53.876	118.320
UC-C-L-3	54	206	803	3.854	14.700	57.397	137.801
UC-C-L-4	54	206	803	3.854	14.700	57.397	137.801
UC-D-1	11	39	152	606	2.258	8.705	26.506
UC-D-2	14	54	208	876	3.261	12.573	35.058
UC-D-3	59	225	878	4.162	15.868	61.942	140.337

Setting	Accuracy [deviation in OFV] 1	2	3	4	5	6	7
COP-C	2,75%	2,93%	3,17%	2,86%	3,15%	3,36%	2,87%
COP-CL	2,74%	2,87%	3,11%	2,85%	3,14%	3,35%	2,86%
COP-B	2,57%	2,81%	2,46%			3,08%	2,64%
COP-BI	2,75%	2,89%	3,12%	2,86%	3,15%		2,88%
ML	2,75%	2,89%	3,16%	2,86%	3,15%	3,36%	2,87%
ST-MD	2,75%	2,93%	3,17%	2,86%	3,15%	3,36%	2,87%
ST-RA	2,37%	2,40%	2,40%	2,48%	2,55%	2,52%	2,48%
DYN	2,41%	2,28%	2,49%	3,07%	2,96%	2,96%	3,16%
TRANS	2,42%	2,60%	2,85%	2,23%	2,51%	2,72%	1,31%
UC-C-1	2,15%	2,14%	2,26%	1,85%	1,64%	1,69%	1,69%
UC-C-L-1	1,81%	1,81%	1,94%	1,24%	1,05%	1,10%	2,44%
UC-C-2	2,37%	2,40%	2,39%	2,48%	2,55%	2,52%	2,48%
UC-C-L-2	2,04%	2,07%	2,08%	1,86%	1,93%	1,91%	1,53%
UC-C-3	1,95%	1,86%	0,83%	1,66%	1,36%	0,87%	1,51%
UC-C-4	1,89%	1,75%	0,63%	1,53%	1,16%	0,67%	1,37%
UC-C-L-3	1,62%	1,53%	0,47%	1,04%	0,77%	0,19%	2,70%
UC-C-L-4	1,56%	1,43%	0,27%	0,91%	0,57%	0,00%	2,83%
UC-D-1	2,08%	1,95%	2,17%	2,43%	2,30%	2,30%	1,08%
UC-D-2	2,06%	1,88%	2,13%	2,42%	2,29%	2,29%	1,07%
UC-D-3	1,27%	0,54%		1,24%	0,60%		2,39%

Quantils: < 25 % < 50 % < 75 % < 100 %

Not solved Benchmark

Table 9: Results for 20 technological settings with seven spatial settings each for (1) the complexity measure solving time [s], (2) the complexity measure model size [Million tableau entries], and (3) the accuracy measure deviation in objective function value (OFV)

References

[1] V. Ş. Ediger, “An integrated review and analysis of multi-energy transition from fossil fuels to renewables,” *Energy Procedia*, vol. 156, pp. 2–6, Jan. 2019.

[2] P. Lopion, P. Markewitz, M. Robinius, and D. Stolten, “A review of current challenges and trends in energy systems modeling,” *Renewable and Sustainable Energy Reviews*, vol. 96, pp. 156–166, Nov. 2018.

[3] W. Schellong, *Analyse und Optimierung von Energieverbundsystemen [Analysis and Optimization of Energy Network Systems]*. Berlin, Heidelberg: Springer Vieweg, 2016.

[4] N. E. Koltsaklis and A. S. Dagoumas, “State-of-the-art generation expansion planning: A review,” *Applied Energy*, vol. 230, pp. 563–589, Nov. 2018.

[5] L. Gacitua *et al.*, “A comprehensive review on expansion planning: Models and tools for energy policy analysis,” *Renewable and Sustainable Energy Reviews*, vol. 98, pp. 346–360, Dec. 2018.

[6] J. DeCarolis *et al.*, “Formalizing best practice for energy system optimization modelling,” *Applied Energy*, vol. 194, pp. 184–198, May 2017.

[7] E. Trutnevyte, “Does cost optimization approximate the real-world energy transition?,” *Energy*, vol. 106, pp. 182–193, Jul. 2016.

[8] S. Nagl, M. Fürsch, and D. Lindenberger, “The Costs of Electricity Systems with a High Share of Fluctuating Renewables: A Stochastic Investment and Dispatch Optimization Model for Europe,” *The Energy Journal*, vol. 34, no. 4, pp. 151–179, 2013.

[9] B. Palmintier and M. Webster, “Impact of unit commitment constraints on generation expansion planning with renewables,” in *2011 IEEE Power and Energy Society General Meeting*, 2011, pp. 1–7.

[10] T. Breuer *et al.*, “Optimizing Large-Scale Linear Energy System Problems with Block Diagonal Structure by Using Parallel Interior-Point Methods,” in *Operations Research Proceedings 2017*, Berlin, 2018, pp. 641–647.

[11] M. Mitchell, *Complexity: A Guided Tour*. Oxford, New York: Oxford University Press, 2009.

[12] L. Chwif, M. R. P. Barretto, and R. J. Paul, “On simulation model complexity,” in *2000 Winter Simulation Conference Proceedings (Cat. No.00CH37165)*, Orlando, FL, USA, 2000, vol. 1, pp. 449–455.

[13] P. P. Craig, A. Gadgil, and J. G. Koomey, “What can History Teach Us? A Retrospective Examination of Long-Term Energy Forecasts for the United States,” *Annu. Rev. Energy. Environ.*, vol. 27, no. 1, pp. 83–118, Nov. 2002.

[14] R. J. Brooks and A. M. Tobias, “Choosing the best model: Level of detail, complexity, and model performance,” *Mathematical and Computer Modelling*, vol. 24, no. 4, pp. 1–14, Aug. 1996.

[15] D. García-Callejas and M. B. Araújo, “The effects of model and data complexity on predictions from species distributions models,” *Ecological Modelling*, vol. 326, pp. 4–12, Apr. 2016.

[16] S. Venkataraman and R. T. Haftka, “Structural optimization complexity: what has Moore’s law done for us?,” *Structural and Multidisciplinary Optimization*, vol. 28, no. 6, pp. 375–387, Dec. 2004.

[17] R. Orth, M. Staudinger, S. I. Seneviratne, J. Seibert, and M. Zappa, “Does model performance improve with complexity? A case study with three hydrological models,” *Journal of Hydrology*, vol. 523, pp. 147–159, Apr. 2015.

[18] C. S. E. Bale, L. Varga, and T. J. Foxon, “Energy and complexity: New ways forward,” *Applied Energy*, vol. 138, pp. 150–159, Jan. 2015.

[19] S. Ikeda and R. Ooka, “Metaheuristic optimization methods for a comprehensive operating schedule of battery, thermal energy storage, and heat source in a building energy system,” *Applied Energy*, vol. 151, pp. 192–205, Aug. 2015.

[20] I. J. Martinez-Moyano, "Documentation for model transparency," *System Dynamics Review*, vol. 28, no. 2, pp. 199–208, 2012.
[21] F. Scheller and T. Bruckner, "Energy system optimization at the municipal level: An analysis of modeling approaches and challenges," *Renewable and Sustainable Energy Reviews*, vol. 105, pp. 444–461, May 2019.
[22] M.-H. Lin, J. G. Carlsson, D. Ge, J. Shi, and J.-F. Tsai, "A Review of Piecewise Linearization Methods," *Mathematical Problems in Engineering*, vol. 2013, pp. 1–8, 2013.
[23] C. Milan, M. Stadler, G. Cardoso, and S. Mashayekh, "Modeling of non-linear CHP efficiency curves in distributed energy systems," *Applied Energy*, vol. 148, pp. 334–347, Jun. 2015.
[24] L. Kotzur, P. Markewitz, M. Robinius, and D. Stolten, "Impact of different time series aggregation methods on optimal energy system design," *Renewable Energy*, vol. 117, pp. 474–487, Mar. 2018.
[25] J. F. Marquant, G. Mavromatidis, R. Evins, and J. Carmeliet, "Comparing different temporal dimension representations in distributed energy system design models," *Energy Procedia*, vol. 122, pp. 907–912, Sep. 2017.
[26] A. Pollok and D. Bender, "Using multi-objective optimization to balance system-level model complexity," in *Proceedings of the 6th International Workshop on Equation-Based Object-Oriented Modeling Languages and Tools - EOOLT '14*, Berlin, Germany, 2014, pp. 69–78.
[27] P. Fritzson, "Modelica—A language for equation-based physical modeling and high performance simulation," in *Applied Parallel Computing Large Scale Scientific and Industrial Problems*, 1998, pp. 149–160.
[28] Z. Sun *et al.*, "Simple or complicated agent-based models? A complicated issue," *Environmental Modelling & Software*, vol. 86, pp. 56–67, Dec. 2016.
[29] P. Voll, "Automated optimization-based synthesis of distributed energy supply systems (Doctoral dissertation)," RWTH Aachen, Aachen, 2014.
[30] S. M. Manson, "Simplifying complexity: a review of complexity theory," *Geoforum*, vol. 32, no. 3, pp. 405–414, Aug. 2001.
[31] J. Ladyman, J. Lambert, and K. Wiesner, "What is a complex system?," *Euro Jnl Phil Sci*, vol. 3, no. 1, pp. 33–67, Jan. 2013.
[32] J. F. DeCarolis, S. Babaee, B. Li, and S. Kanungo, "Modelling to generate alternatives with an energy system optimization model," *Environmental Modelling & Software*, vol. 79, pp. 300–310, May 2016.
[33] K.-K. Cao, A. Gleixner, and M. Miltenberger, "Methoden zur Reduktion der Rechenzeit linearer Optimierungsmodelle in der Energiewirtschaft - Eine Performance-Analyse," in *EnInnov 2016: 14. Symposium Energieinnovation 2016*, 2016.
[34] S. Han, W. Won, and J. Kim, "Scenario-based approach for design and comparatively analysis of conventional and renewable energy systems," *Energy*, vol. 129, pp. 86–100, Jun. 2017.
[35] A. Sarid and M. Tzur, "The multi-scale generation and transmission expansion model," *Energy*, vol. 148, pp. 977–991, Apr. 2018.
[36] E. Merkel, R. McKenna, and W. Fichtner, "Optimisation of the capacity and the dispatch of decentralised micro-CHP systems: A case study for the UK," *Applied Energy*, vol. 140, pp. 120–134, Feb. 2015.
[37] S. Pfenninger, "Dealing with multiple decades of hourly wind and PV time series in energy models: A comparison of methods to reduce time resolution and the planning implications of inter-annual variability," *Applied Energy*, vol. 197, pp. 1–13, Jul. 2017.

[38] L. Kotzur, P. Markewitz, M. Robinius, and D. Stolten, “Time series aggregation for energy system design: Modeling seasonal storage,” *Applied Energy*, vol. 213, pp. 123–135, Mar. 2018.

[39] H. Teichgraeber and A. R. Brandt, “Clustering methods to find representative periods for the optimization of energy systems: An initial framework and comparison,” *Applied Energy*, vol. 239, pp. 1283–1293, Apr. 2019.

[40] B. S. Palmintier and M. D. Webster, “Heterogeneous Unit Clustering for Efficient Operational Flexibility Modeling,” *IEEE Transactions on Power Systems*, vol. 29, no. 3, pp. 1089–1098, 2013.

[41] B. S. Palmintier and M. D. Webster, “Impact of Operational Flexibility on Electricity Generation Planning With Renewable and Carbon Targets,” *IEEE Transactions on Sustainable Energy*, vol. 7, no. 2, pp. 672–684, Apr. 2016.

[42] G. Morales-Espana and D. A. Tejada-Arango, “Modelling the Hidden Flexibility of Clustered Unit Commitment,” *IEEE Transactions on Power Systems*, pp. 1–1, 2019.

[43] Z. Duan *et al.*, “An MILP Method for Design of Distributed Energy Resource System Considering Stochastic Energy Supply and Demand,” *Energies*, vol. 11, no. 1, p. 22, Jan. 2018.

[44] L. Yang, C. Zhang, J. Jian, K. Meng, Y. Xu, and Z. Dong, “A novel projected two-binary-variable formulation for unit commitment in power systems,” *Applied Energy*, vol. 187, pp. 732–745, Feb. 2017.

[45] N. Alguacil and A. J. Conejo, “Multiperiod optimal power flow using Benders decomposition,” *IEEE Transactions on Power Systems*, vol. 15, no. 1, pp. 196–201, Feb. 2000.

[46] H. R. Abdolmohammadi and A. Kazemi, “A Benders decomposition approach for a combined heat and power economic dispatch,” *Energy Conversion and Management*, vol. 71, pp. 21–31, Jul. 2013.

[47] T. Falke, S. Krengel, A.-K. Meinerzhagen, and A. Schnettler, “Multi-objective optimization and simulation model for the design of distributed energy systems,” *Appl. Energy*, vol. 184, pp. 1508–1516, Dec. 2016.

[48] I. Keppo and M. Strubegger, “Short term decisions for long term problems – The effect of foresight on model based energy systems analysis,” *Energy*, vol. 35, no. 5, pp. 2033–2042, May 2010.

[49] E. M. L. Beale and J. A. Tomlin, “Special facilities in a general mathematical programming system for non-convex problems using ordered sets of variables,” in *Proceedings of the Fifth International Conference on Operational Research*, London, 1970, pp. 447–454.

[50] J. Nocedal and S. Wright, *Numerical Optimization*, 2nd ed. New York: Springer-Verlag, 2006.

[51] G. L. Nemhauser and L. A. Wolsey, *Integer and Combinatorial Optimization*. New York, NY, USA: Wiley-Interscience, 1988.

[52] H. P. Williams, *Model Building in Mathematical Programming*, 5th ed. John Wiley & Sons Ltd, 2013.

[53] J. F. Benders, “Partitioning procedures for solving mixed-variables programming problems,” *Numer. Math.*, vol. 4, no. 1, pp. 238–252, Dec. 1962.

[54] M. Aghighi, “Computational Complexity of some Optimization Problems in Planning (Doctoral dissertation),” p. 57, 2017.

[55] T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, “Chapter 2 ‘Getting Started,’” in *Introduction To Algorithms*, MIT Press, 2001.

[56] W. A. J. Kosmala, *A friendly introduction to analysis: single and multivariable*, 2nd ed. Upper Saddle River, N.J.: Pearson Prentice Hall, 2004.

[57] A. S. Tehrani, Haiying Cao, S. Afsardoost, T. Eriksson, M. Isaksson, and C. Fager, “A Comparative Analysis of the Complexity/Accuracy Tradeoff in Power Amplifier Behavioral Models,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 58, no. 6, pp. 1510–1520, Jun. 2010.
[58] H.-K. Ringkjøb, P. M. Haugan, and I. M. Solbrekke, “A review of modelling tools for energy and electricity systems with large shares of variable renewables,” *Renewable and Sustainable Energy Reviews*, vol. 96, pp. 440–459, Nov. 2018.
[59] W. Schoutens, E. Simons, and J. Tistaert, “A Perfect Calibration! Now What?,” *Wilmott Magazine*, 2004.
[60] Y. Censor, “Pareto optimality in multiobjective problems,” *Appl Math Optim*, vol. 4, no. 1, pp. 41–59, Mar. 1977.
[61] G. Erichsen, T. Zimmermann, and A. Kather, “Effect of Different Interval Lengths in a Rolling Horizon MILP Unit Commitment with Non-Linear Control Model for a Small Energy System,” *Energies*, vol. 12, no. 6, p. 1003, Jan. 2019.
[62] K. Van den Bergh, K. Bruninx, E. Delarue, and W. D’haeseleer, “A Mixed-Integer Linear Formulation of the Unit Commitment Problem,” K.U. Leuven Energy Institute, WP EN2014-07, 2014.
[63] S. Pfenninger and J. Keirstead, “Renewables, nuclear, or fossil fuels? Scenarios for Great Britain’s power system considering costs, emissions and energy security,” *Applied Energy*, vol. 152, pp. 83–93, Aug. 2015.
[64] J. Meus, K. Poncelet, and E. Delarue, “Applicability of a Clustered Unit Commitment Model in Power System Modeling,” *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 2195–2204, Mar. 2018.
[65] ENTSO-E, “[dataset] ENTSO-E transparency platform,” *ENTSO-E transparency platform*, 2018. [Online]. Available: transparency.entsoe.eu. [Accessed: 26-Feb-2019].
[66] LAK, “[dataset] Endenergieverbrauch nach Energieträgern [Final Energy Consumption by Energy Source],” *Länderkreis Energiebilanzen [Federal Working Group on Energy Balances]*, 2015. [Online]. Available: http://www.lak-energiebilanzen.de/endenergieverbrauch-nach-energietraegern-aktuell/. [Accessed: 26-Feb-2019].
[67] Statistische Ämter des Bundes und der Länder [Federal and State Statistical Offices], “[dataset] Volkswirtschaftliche Gesamtrechnungen der Länder [National Accounts of the Federal States],” *Statistikportal*, 2016. [Online]. Available: https://www.statistik-bw.de/VGRdL/tbls/?rev=RV2014&lang=de-DE#RV2014KR. [Accessed: 26-Feb-2019].
[68] C. Matke, W. Medjroubi, and D. Kleinhans, “SciGRID - An Open Source Reference Model for the European Transmission Network (v0.2),” *SciGRIDv0.2*, 2016. [Online]. Available: www.scigrid.de.
[69] TrustChain e.G., “[dataset] Postal Codes in JSON, XML and CSV format,” 2018. [Online]. Available: https://github.com/TrustChainEG/postal-codes-json-xml-csv, licensed under Creative Commons Attribution 4.0 International. [Accessed: 26-Feb-2019].
[70] BNetzA, “[dataset] Kraftwerksliste der Bundesnetzagentur - Stand: 10.05.2016 [Power Plant List of the Federal Network Agency - Status: 10.05.2016],” *Bundesnetzagentur [Federal Network Agency]*, 2016. [Online]. Available: https://www.bundesnetzagentur.de/SharedDocs/Downloads/DE/Sachgebiete/Energie/Unternehmen_Institutionen/Versorgungssicherheit/Erzeugungskapazitaeten/Kraftwerksliste/Kraftwerksliste_2015.html. [Accessed: 26-Feb-2019].
[71] UBA, “[dataset] Datenbank ‘Kraftwerke in Deutschland’ [Database ‘Power plants in Germany’],” *Umweltbundesamt [Federal Environment Agency]*, 2018. [Online]. Available: http://www.umweltbundesamt.de/dokument/datenbank-kraftwerke-in-deutschland. [Accessed: 26-Feb-2019].

[72] P. Mondal and S. Ghosh, “Exergo-economic analysis of a 1-MW biomass-based combined cycle plant with externally fired gas turbine cycle and supercritical organic Rankine cycle,” *Clean Techn Environ Policy*, vol. 19, no. 5, pp. 1475–1486, Jul. 2017.
[73] S. Khanmohammadi, A. R. Azimian, and S. Khanmohammadi, “Exergy and exergo–economic evaluation of Isfahan steam power plant,” *International Journal of Exergy*, vol. 12, no. 2, pp. 249–272, Jan. 2013.
[74] M. M. Martínez, “Optimization models and techniques for implementation and pricing of electricity markets (Doctoral dissertation),” University of Waterloo, 2000.
[75] W. Ströbele, W. Pfaffenberger, and M. Heuterkes, *Energiewirtschaft: Einführung in Theorie und Politik*, vol. 3. Walter de Gruyter, 2012.
[76] A. Gleixner *et al.*, “The SCIP Optimization Suite 5.0,” Zuse Institute Berlin, Berlin, ZIB Report 17–12, 2017.
[77] H. K. Mehta, *Mastering Python scientific computing: a complete guide for Python programmers to master scientific computing using Python APIs and tools*. Birmingham, Mumbai: Packt Publishing, 2015.
[78] C. Bredtmann, J. Kellermann, and A. Moser, “Vergleich kommerzieller und freier Solver bei der Durchführung von Netzbetriebssimulationen [Comparison of commercial and free solvers in the execution of network operation simulations],” in *15. Symposium Energieinnovation*, Graz, 2018.