Stochastic Modelling: Energy and Fuel-Switching Pricing Using Levy Processes - An Application to Canadian and North American Data

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Abstract

The Paris agreement in 2016 marks a global effort to limit the increase in temperature. In that spirit, the Federal Government of Canada introduced a carbon tax to reduce greenhouse gas emissions. The main goal of this paper is to define the correct approach to carbon pricing. Following the method, introduce by Goutte and Chevalier (2015), we define the carbon price as the necessary tax to incite electricity producers to switch from coal to natural gas. In addition, we consider the case of switching from natural gas to wind as a potential approach. After reviewing the two methods, we model prices under three stochastic procedures: pure-jump Lévy process, Lévy Normal and Heston model. Finally, we generalize our empirical technique to oil, natural gas and coal individually. The main finding of this article is that the Lévy process outperforms the Lévy Normal and Heston as it is able to take into account the jumpy and volatile nature of energy prices.

Key words: Energy Economics, Stochastic Processes

1 Introduction

In October 2016, the federal government of Canada published "The pan-Canadian approach to pricing Carbon Pollution". The main goal of this act is to reduce greenhouse gas (GHG) emissions by taxing fossil fuels responsible for releasing carbon in the atmosphere. Over the years, fossil fuels, namely, coal, natural gas, and oil, have been seen as the main cause of temperature disruptions and extreme weather events. Climate specialists predict that temperatures could rise up to 5 degrees Celsius in 2100 (Chesney and Taschini, 2012). As fossil fuels contain carbon, once burnt, they allow energy to be generated, which in turn is important for health, education, political power and economic status (Sneideman, 2015). The Canadian consensus is in line with the climate agreement reached in Paris in 2016 where countries have agreed to a common effort to limit temperature increase to 1.5 degrees. Furthermore, developed countries are to provide help during extreme weather events and slow-onset such as the sea level rise. Finally, financial support should be given to developing countries in order to invest in clean energy (UNFCC, 2016).

If a consensus has been reached regarding efforts to protect the earth, the implementation of a carbon pricing scheme divides the fiercest economists, especially in Canada. Two systems are currently in place: cap and trade, and taxation. On the one hand, a cap and trade give an initial number of permits to emit CO2 to a company based on its activity. If an enterprise emits less than what the number of permits allows them, they can freely trade the number of permits in excess to another entity that wishes to pollute more. Economically more efficient, this approach gives an incentive to a company to reduce its pollution level in order to sell their permit. On the other hand, a taxation approach gives the right to a company to emit a certain level of CO2. If the firm releases more CO2, it has to pay a tax. As one can see, firms do not have an incentive to pollute less than what the limit prevails. In Canada, Quebec and Ontario adopted a cap and trade system, where the carbon price is $18 per tonne on average for the Quebec Province (Tombe and Rivers, 2017). However, the western provinces of BC and Alberta opted for a taxation system. The current carbon price in Alberta is $30 per tonne. The introduction of carbon pricing is said to have different impacts among regions. A case study of the province of Saskatchewan, whose economy is based on the extraction of natural resources, shows that GHG emissions are reduced but its economy is bound to shrink as the fuel-switching oppor-

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tunities are rare (Liu et al, 2018). However, the BC example shows promising results. Introducing its carbon tax in 2008, Yamazaki (2017) found evidence that the system in place proved to be beneficial for the employment rate of the province.

This paper focuses on Alberta who introduced the carbon tax in 2017. Alberta is currently the biggest coal producer alongside British Columbia as they provide 85% of Canadian coal. The use of this fossil fuel currently generates 10% of the country's electricity. Moreover, the province supplies 71% of Canada's natural gas (NRCAN, 2018). In 2018, the carbon tax increased by 50% to reach 30$ per tonne. Households electricity bills are expected to rise by 150$ (Tombe, 2017). The main concern raised by economists is that the tax is going to affect poor households, and this, in turn, could lead to an increase in inequalities (Ambasta and Buonocore, 2018). Hence, a correct approach to carbon pricing is essential for the Albertan economy and its residents.

In this paper, we introduce an approach to carbon pricing based on the European energy market experience. Following the method introduced by Goutte and Chevallier (2015), we define the carbon price as the necessary price to incite companies to switch from coal to natural gas. Since the latter is less carbon intensive, this measure would considerably reduce GHG emissions. Besides, we also examine the case of switching from natural gas to wind. The economic content of this paper gives a detailed explanation of the fuel-switching and energy-switching processes. Moreover, we intend to generalize our statistical approach to the North American market and other energy indicators. In addition to the fuel-switching and energy-switching prices, we look at coal, natural gas, and oil individually. In order to model the prices, we consider three types of stochastic models: Lévy Normal Inverse Gaussian (NIG) process, Lévy Normal, and the Heston model.

The paper is structured in the following way. Section 2 provides the necessary economic background to understand how the fuel-switching price is defined. Section 3 presents the data used and gives the first insight into the Albertan energy market. Section 4 is concerned with the methodology and can be dissected into two parts: stochastic modeling and parameters estimation. Section 5 shows the empirical results found. Section 6 discusses the potential shortcomings of the paper and topics for further research and summarizes the main results.

2 Energy Economics and Energy-Switching

Energy markets depend on micro and macroeconomic factors and influence fuel-switching and energy-switching. The aim of this section is to first give the dynamic driving energy prices. Secondly, the notion of energy-switching is defined as well as the necessary conditions for it to happen and the potential problems arising from it. Thirdly, the carbon pricing formula is presented and reveals when companies have an incentive to pass from coal to natural gas (or natural gas to wind) and vice-versa based on current market conditions. Finally, we examine the factors influencing the price of wind.

Prices in the energy sector depend on political decisions and economic aspects. Competition among fossil fuel users and from alternative sources to generate energy, such as renewables, is a key factor in defining prices. Indeed, energy markets are often controlled by a monopolist who has the power to dictate prices. Moreover, subsidies given by governments to clean-technologies can play an important role in the competitiveness of the industry. In addition to the competition facet, national allocation plans, which covers the initial number of permits (CO2 allowances) and a penalty level, are identified as the main cause of price jumps. Furthermore, the volatility of the price of fossil fuels is also an element to take into consideration. In fact, coal prices are generally more stable than natural gas prices and, consequently, are more attractive for a company looking to reduce risks. Other variables potentially influencing prices are weather conditions and economic growth (Sjim et al, 2006; Seifert et al, 2008; Carmona et al, 2009).

Another influence omitted from the list above is fuel-switching, which represents the possibility to pass from a coal-fired plant to a natural gas plant, and vice-versa. Coal is generally cheaper and thus preferred by companies, even though it emits more CO2. Therefore, in order for a switch to happen two conditions must be met. First, the carbon price (tax or current permit price) must be high enough and natural gas price low enough. Since natural gas emits less CO2, a high carbon price favors its use. Second, there has to be the physical possibility to switch. During the winter season, the demand for electricity is typically higher than in the summer, and it is not unlikely that all plants are working at their maximum capacity, regardless of the type of fossil fuels used (Delarue, D'haeseleer, 2007). The fuel-switching process is a good start to model carbon price since traditional abatement measures tend to invite producers to use cleaner energy than coal. However, as noted by Chesney and Taschini (2012), fuel purchasers tend to sign contracts with a long maturity and this impedes the fuel-switching process to be fully flexible. Consequently, this paper chooses to consider monthly fuel prices, rather than daily.

Electricity prices depend on the physical capacity to generate power, the presence of potential substitute and other economic factors. Therefore, an adequate formula must take into account the various aspects mentioned so far. This paper follows the method defined by Chevallier and Goutte (2015). Prices are defined by the marginal generation of technology and are expressed as the ratio between the fuel cost, FC, and the plant efficiency, η, which represents the necessary amount of energy needed to produce electricity:

$$MC = \frac{FC}{\eta}$$

Introducing a price on carbon requires the equation above to be revised. Considering that fossil fuels have a different impact on the environment, an emission factor, EF, based on C02 intensity is added as well as an emission cost, EC, per unit of carbon emitted. The revised formula reads:

$$MC = \frac{FC}{\eta} + \frac{EF}{\eta}EC$$

Fuel-switching occurs if the use of one fossil fuel to generate energy is cheaper than the other option. Therefore, by equalizing the marginal costs, one can define the minimum carbon price necessary for a switch to occur. Indeed, the only factor which is common to both fossil fuels is EC.

$$EC_{switch} = \frac{\eta_{coal}FC_{gas} - \eta_{gas}FC_{coal}}{\eta_{gas}EF_{coal} - \eta_{coal}EF_{gas}}$$

If the carbon tax defined by the Albertan government is lower than this price, then coal plants are said to be more profitable than gas plants.

The idea behind energy-switching based on wind follows the same formula. However, the factors influencing wind prices are quite different. Indeed, the use of wind depends on seasonal factor and its price depends on technological aspect, such as electricity storage. The Canadian Wind Energy Association (CanWea) provides further explanation regarding this topic. Moreover, wind is discussed in greater details in the next section.

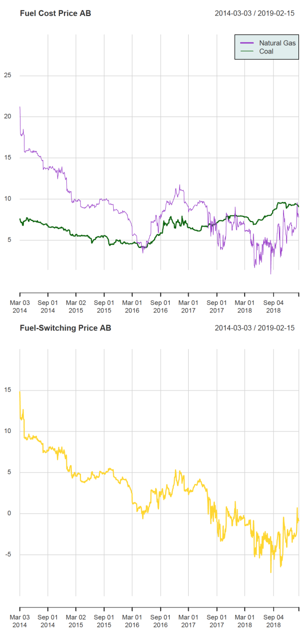
3 Data

The data used in this project were gathered from various sources, such as NRG Stream, Bloomberg, Energy Information Association (EIA), Market Insider, Jem Energy, the city of Winnipeg and CanWea. This section provides insight into the distribution and evolution of energy prices over time, as well as the justification for the time period considered.

Following the formula for the fuel-switching (energy-switching) prices from the previous sections, the different variables employed were retrieved from various data sources. When fuel-switching is considered, we need data regarding the fuel cost, efficiency parameter and emission factor as mentioned in the previous section. Prices of coal are in \$/tonne and prices of natural gas in \$/MMbtu were retrieved from NRG Stream (Alberta data), EIA (natural gas North America data) and Market Insider (coal North America data). The emission factor component, EF, is expressed in kgCO2eq/MWhp. Financial data from the city of Winnipeg show that EF is equal to 210 for natural gas and 320 for coal. Similar to previous findings, coal is more harmful to the environment than natural gas. A 2004 study from JEM Energy calculated the efficiency of the coal and natural gas plants in Alberta. The average efficiency for a coal and natural gas plant is respectively, 32.6% and 31.1% (32% and 43% respectively for North America, according to EIA)

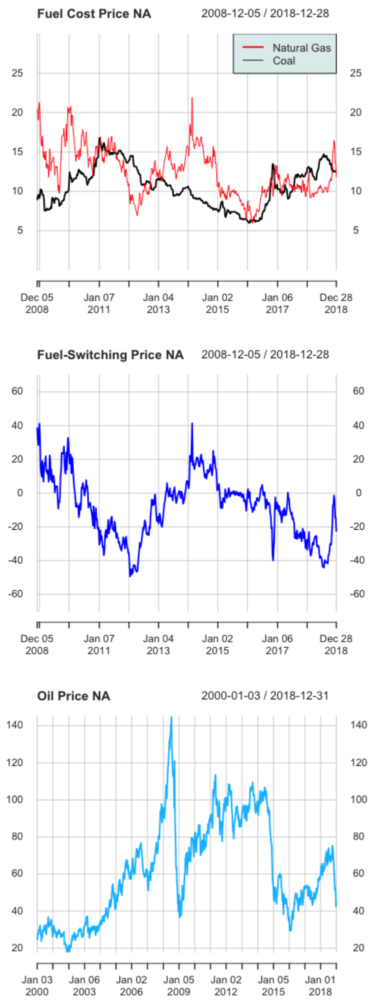
In order to compute the EC price, the data were changed from \$/MMbtu and \$/tonne to \$/Mwh. Furthermore, weekly data and daily data were chosen as opposed to monthly data. Fuel-switching is technically more likely to happen on a monthly basis, however, since we failed to obtain large time series data for the energy markets, we opted for a weekly approach. The next figures below, represent the evolution of coal, natural gas, oil (Bloomberg data) and fuel switching for both the Albertan and North American markets.

Fig. 1. Alberta Market



The results for the North American market were obtained using Bloomberg data. In recent years, the prices for coal and natural gas have become extremely close to natural gas being even cheaper at times. Consequently, it is not surprising that we observe the fuel-switching price to have gone negative. The main implication is that pricing carbon using fuel-switching is not appropriate anymore. Regarding oil, we note that the financial crisis may have triggered a high-spike in its price. By looking at the three figures, we conclude that energy prices are characterized by high-spike and quick mean reversion, which justify our approach to use pure jumps methods.

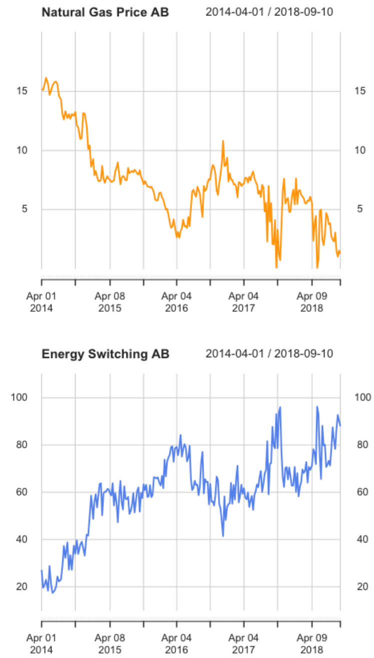
Fig. 2. North American Market



Additionally, it is apparent that prices go through periods of calm and stress, this, in turn, could imply that a stochastic volatility model, such as Heston model, yields better results than a classic geometric Brownian motion. Finally, it is indisputable that fuel-switching has become obsolete, therefore, we decide to model carbon under our energy-switching approach.

In recent years, renewable energies, wind in particular, have become an important source of electricity generation. A 2016 study by the Canadian Wind Energy Association (CANWEA), shows that wind power accounts for 50% of Denmark's electricity generation system and is the largest source of new Energy in Canada. The emission factor, associate to wind is equal to zero and it's efficiency ranges from 32-37% according to the EIA. Moreover, the cost of Wind is estimated to lie somewhere between 37.5$ and 42.5$ per MWh. Since data regarding wind cost is difficult to estimate, we generated a random uniformly distributed process to obtain the price of wind. The next figure shows the necessary price to switch from natural gas to wind. If the carbon tax is under the line, then natural gas plants are said to be more efficient than wind plants. The energy-switching price is quite high and reflects the current low price of natural gas in Alberta.

Fig. 3. Energy Switching based on wind for Alberta



So far, our results have indicated that fuel-switching, considering the large drop in the natural gas price, is no longer an adequate approach. Moreover, the prices are characterized by jumps and mean-reversion and this confirms what has previously been observed in the literature. Therefore, the last step prior to stochastic modeling is to determine the empirical distribution of our data.

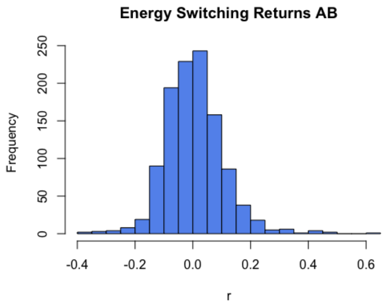
The distribution of returns is a good indicator of the problems that can arise when simulating prices under a geometric Brownian approach. Financial data, as illustrated by oil and energy-switching, depart from a Normal distribution. Indeed, extreme returns are more likely to happen than what the Normal predicts. Moreover, we notice that returns are generally skewed. Hence, the use of a NIG may be a more appropriate approach when simulating energy prices.

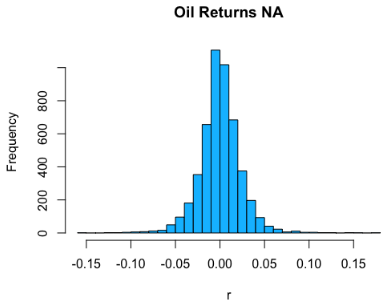
4 The stochastic Model

In this part, we are considering a panel of continuous mean reverting stochastic process and Heston model. The fuel-switching price

Let (ω, F, P) be a probability space. In this paper, all the stochastic models are under this probability space.

Fig. 4. Empirical Distribution of Returns





We now present three models and compare each of them to obtain a best overall fit to the fuel-switching price.

4.1 Mean-reverting process

Definition 1: A Lévy process $\{X_t\}_{t\geq 0}$ is a stochastic process that it satisfies following properties:
1.$X_0 = 0$.
2.For any $s > 0$ and $t > 0$, we have that $X_{t+s} - X_t$ has the same distribution with X_s. i.e. It has stationary increments.
3. For $0 \leq t_0 < t_1 < ... < t_n$, $X_{t_i} - X_{t_{i-1}}$ are independent for all i. i.e. It has independent increments.
4. The path of a Lévy process are right continuous and admit left limit. i.e. X_t has Cadlag path.

Next, we will give two mean reverting process, one is a continuous process with a browian motion and the other one is Lévy-driven Ornstein-Uhlenbeck processes.

Definition 2: For all $t \in [0, T]$, a continuous mean reverting process with Brownian motion is a stochastic process (X_t) which is a solution of the stochastic defferential equation:

$$dX_t = (\theta - X_t)dt + \sigma dW_t$$

where κ,θ,σ are constants and W_t is a standard Brownian motion.

Remark 1: In this model, κ denotes the mean-reverting rate, θ denotes the long-run mean and σ denotes the volatility of (X_t).

Definition 3: The continuous process mean reverting process with pure Lévy jump process writes:

$$dX_t^L = (\theta - X_t)dt + \sigma_L dL_t$$

with parameters κ,θ in R, $\sigma_m \in R^+$ and L_t is a Lévy process. The solution to this stochastic differential equation is called Lévy-driven Ornstein-Uhlenbeck processes.

Remark 2: The Lévy process L can follow different kinds of distributions for example the Variance Gamma distribution. In this paper we assume that it follows a Normal Inverse Gaussian(NIG) distribution. This family of distribuiton was introduced by Barndorff-Nielsen(1998) and it is a continuous probability distribution that is defined as the Normal variance-mean mixture where the mixing density is the inverse Gaussian distribution.

Definition 4:
Let $\delta > 0$,$\alpha \geq 0$ and $\gamma \geq 0$, then the probability density function of Normal Inverse Gaussian distribution is :

$$\frac{\alpha\delta}{\pi} exp(\delta\sqrt{\alpha^2 - \beta^2} + \beta(x - \mu)) \frac{K_1(\alpha\delta\sqrt{1 + (x - \mu)^2/\delta^2})}{\sqrt{1 + (x - \mu)^2/\delta^2}}$$

where the K_v is a Bessel function of the third kind with index v and it can be represented with the following integral:

$$K_v(z) = \frac{1}{2}\int_0^\infty y^{(v-1)} exp(-\frac{1}{2}z(y + y^{-1}))dy$$

Also, for a given real v, K_v satisfies the differential equation given by

$$x^2y'' + xy' - (x^2 + v^2)y = 0$$

4.2 Estimations of mean reverting stochastic process

In this section we consider a two step parameter estimation method for this model. In Goutte's paper he developed a least square method which minimize the empirical variance to obtain the parameter for Brownian motion and he used a constrained maximum likelihood method for estimating the NIG random variable.

Before estimating parameters, we first need to discretization the model. In practice, we observe the price at fixed times $0 = t_0 < t_1 < ... < t_n = T$, with $\Delta t = t_{k+1} - t_k$

constant. Thus we can first solve the stochastic differential equation and discretize the solution by:

$$X_{t_{k+1}} = X_{t_k} e^{-\kappa\Delta t} + \int_{t_k}^{t_{k+1}} \kappa\theta e^{-\kappa(t_{k+1}-s)} ds + \int_{t_k}^{t_{k+1}} \sigma e^{-\kappa(t_{k+1}-s)} dL_s$$

Rearrange the solution, we obtain:

$$X_{t_{k+1}} - X_{t_k} = m - aX_{t_k} + s\epsilon_{t_k}$$

with $m = (1 - e^{-\kappa\Delta t})\theta$, $a = 1 - e^{-\kappa\Delta t}$ and $s\epsilon_k = \int_{t_k}^{t_{k+1}} \sigma e^{-\kappa(t_{k+1}-s)} dL_s$

- If the model is mean reverting process with Brownian motion, then the process L is Brownian motion and ϵ_k follows $N(0,1)$.
- If the model is Lévy-driven Ornstein-Uhlenbeck processes, then the process L follows a NIG distribution with expectation 0 and variance 1. Thus we assume that the parameters of NIG distribution in this model are α, β, δ and μ.

Thus, we need to estimate the set of parameters $\{m, a, s, \alpha, \beta, \delta, \mu\}$

4.2.1 Parameter Estimation procedure: step one

We estimate the subset of parameter $\{m, a, s\}$ at first using a least square method that minimize the empirical variance of the noise:

$$Var[s\epsilon] \approx \frac{1}{n} \sum_{k=0}^{n-1} (X_{k+1} - (1+a)X_k - m)^2$$

where n is the amount of data we have.
The solutions to this quadratic problem is:

$$\begin{bmatrix} \hat{m} \\ 1-\hat{a} \end{bmatrix} = (A'A)^{-1}A'B$$

Where A= $\begin{bmatrix} 1 & X_{n-1} \\ ... & ... \\ 1 & X_0 \end{bmatrix}$ and B= $\begin{bmatrix} X_n \\ ... \\ X_1 \end{bmatrix}$.

Thus the estimator of s is directly followed by

$$\hat{s}^2 = \hat{s}^2 Var[\epsilon] = Var[\hat{s}\epsilon] = \frac{1}{n} \sum_{k=0}^{n-1} (X_{k+1} - (1+\hat{a})X_k - \hat{m})^2.$$

4.2.2 Parameter Estimation procedure: step two

In this step we propose a constrained maximum likelihood method to estimate the parameter $\{\alpha, \beta, \delta, \mu\}$. So far we assume that we have n+1 observations (the prices) $(X_0, X_1, ..., X_n)$ such that, for $k = 0, 1, ..., n-1$,

$$\tilde{\epsilon}_k = X_{k+1} - (1-\hat{a})X_k = \hat{m} + \hat{s}\epsilon_k$$

is followed by the non-centered and unNormalized NIG distribution $NIG(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\mu})$. Now, we are willing to estimate these parameters based on the following propositions.

Proposition 1 : Suppose $X_1, X_2, ..., X_n \sim NIG(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\mu})$, then log-likelihood function is given by

$$nlog(\frac{\tilde{\alpha}\tilde{\delta}}{\pi}) + n\tilde{\delta}\tilde{\gamma} + \sum_{k=0}^{n-1} [\tilde{\beta}\tilde{\delta}\tau_k - \log c_k + \log K_1(\tilde{\alpha}\tilde{\delta}c_k)]$$

where $\tau_k = \frac{X_k - \tilde{\mu}}{\delta}$, $c_k = \sqrt{1+\tau_k^2}$, $\tilde{\gamma} = \sqrt{\tilde{\alpha}^2 + \tilde{\beta}^2}$.

Proposition 2: If $X \sim NIG(\alpha, \beta, \delta, \mu)$, then for any $a \in R^+$ and $b \in R$, we have

$$Y = aX + b \sim NIG(\frac{\alpha}{a}, \frac{\beta}{a}, a\delta, a\mu + b)$$

Proposition 3:The first four central moments of the NIG distribution are:

$$m_1 = \mu + \delta\beta\gamma^{-1}, m_2 = \delta\alpha^2\gamma_{-3},$$
$$m_3 = 3\delta\beta\alpha^2\gamma^{-5}, m_4 = 3\delta\alpha^2(\alpha^2 + 4\beta^2)\gamma^{-7}$$

By proposition 1, we can estimate the parameters $\{\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\mu}\}$ by maximize the log likelihood under constrains $\tilde{\gamma} > 0$ and $\tilde{\delta} > 0$. Due to complicated form of the density of NIG distribution, obtain a estimators for our parameters is a difficult task and so we need special numerical method to solve it.

Once the parameter set $\{\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\mu}\}$ has been estimated, we then use proposition 2. We know that

$$\epsilon_k = \frac{\tilde{\epsilon}_k}{\hat{s}} - \frac{\hat{m}}{\hat{s}} \sim NIG(\hat{s}\tilde{\alpha}, \hat{s}\tilde{\beta}, \frac{\tilde{\delta}}{\hat{s}}, \frac{\tilde{\mu} - \hat{m}}{\hat{s}}).$$

So the true estimates of the parameters $\{\alpha, \beta, \delta, \mu\}$ are:

$$\alpha = \hat{s}\tilde{\alpha}, \quad \beta = \hat{s}\tilde{\beta}, \quad \delta = \frac{\tilde{\delta}}{\hat{s}}, \quad \mu = \frac{\tilde{\mu} - \hat{m}}{\hat{s}}.$$

Recall in previous we want the expectation of ϵ_k to be 0 and variance to be 1. Thus according to proposition 3, we also need $E[\epsilon_k] = \mu + \frac{\delta\beta}{\gamma} = 0$ and $Var[\epsilon_k] = \frac{\delta\alpha^2}{\gamma^3} = 1$. Combine with the four equation above, we can conclude that we only have two free parameters (α, β).

At last, since we are using special numerical method to maximize the log-likelihood function, we need to give our method some good initial values. Under this situation, we find out the first to forth sample moments based on X_i and then solve the initial value of these parameters. Now let

$$\mu_1 = \tilde{\mu} + \tilde{\delta}\tilde{\beta}\tilde{\gamma}^{-1}, \quad \mu_2 = \tilde{\delta}\tilde{\alpha}^2\tilde{\gamma}^{-3}$$
$$\mu_3 = 3\tilde{\delta}\tilde{\beta}\tilde{\alpha}^2\tilde{\gamma}^{-5}, \quad \mu_4 = 3\tilde{\delta}\tilde{\alpha}^2\tilde{\alpha}^2 + 4\tilde{\beta}^2\tilde{\gamma}^{-7}$$

where $\mu_k = \frac{1}{n}\sum_{j=0}^{n-1}(\tilde{\epsilon_j} - \bar{X})^k$, $k = 1, 2, 3, ...$ is the kth sample moment.

Using these four equation we obtain,

$$\hat{\tilde{\gamma}} = \frac{3}{\bar{S}\sqrt{3\bar{\gamma}_2 - 5\bar{\gamma}_1{}^2}}, \quad \hat{\tilde{\beta}} = \frac{\bar{\gamma}_1\bar{S}\hat{\tilde{\gamma}}^2}{3}$$
$$\hat{\tilde{\delta}} = \frac{\bar{S}^2\hat{\tilde{\gamma}}^3}{\hat{\tilde{\beta}}^2 + \hat{\tilde{\gamma}}^2}, \quad and \quad \hat{\tilde{\mu}} = \bar{X} - \hat{\tilde{\beta}}\frac{\hat{\tilde{\delta}}}{\hat{\tilde{\gamma}}}$$

where $\bar{X}$ and $\bar{S}$ are the sample mean and variance respectively and $\bar{\gamma}_1 = \frac{\mu_3}{\mu_2^{\frac{3}{2}}}$, $\bar{\gamma}_2 = \frac{\mu_4}{\mu_2^2} - 2$.

4.3 Heston Model

Definition 5: (Heston model) Under the risk-neutural probability measure Q the Heston model is given by:

$$dS(t) = rS(t)dt + \sqrt{V(t)}S(t)dW_s(t)$$
$$dV(t) = \kappa(\theta - V(t))dt + \sigma\sqrt{V(t)}dW_v(t)$$

where $W_s(t)$ and $W_v(t)$ are two Brownian motions with correlation coefficient ρ. We then apply the Euler-Maruyama scheme to the equation above and simulate it. The initial estimates for the volatility equation are derived under an OLS approach. The risk-free rate r is arbitrarily initiated at zero. As starting value of the process we decide to use the mean of the energy price.

Algorithm 1:. Let $\hat{X}$ and $\hat{V}$ denote discrete-time approximations of X and V respectively. The Euler-Maruyama scheme applied to the above equation is given by

$$\hat{X}(h(i)) = \hat{X}(h(i-1)) + (r - \frac{1}{2}\hat{V}(h(i-1)))h + \sqrt{V(h(i-1))}Z_x\sqrt(h)$$

$$\hat{V}(h(i)) = \hat{V}(h(i-1)) + \kappa(\theta - V(h(i-1)))h + \sigma\sqrt{V(h(i-1))}Z_v\sqrt(h)$$

where Z_x and Z_y are standardized Gaussian random variables such that $corr(Z_x, Z_y) = \rho$.

5 Empirical Analysis

This section provides the main findings of this article. First, we begin by comparing the general statistic of our data with the empirical estimation of our parameters. Second, the simulations obtained are displayed. Third, the goodness of fit of the data is assessed.

5.1 Parameter Estimation

Table 1 below presents a summary of the main information regarding energy prices. Table 2 provides the estimated OLS parameter for the Lévy-pure jump and Brownian motion processes. Table 3 displays the estimates using the simulate the NIG.

The data section of this article presented the real price of each energy stock and demonstrated that they were subject to high spikes and jumps. Table 1 confirms our past impression. It is clear that prices of oil vary quite substantially with a minimum value of 17.72\$/barrel and a high of 145.18\$/barrel. This can potentially be explained by the power of cartels to dictate prices. Moreover, the skewness and kurtosis measurement indicate departures from the Normal as previously assumed by looking at the distribution of prices. Indeed, a positive kurtosis means that extreme events are more likely than if the data came from a Normal distribution. Consequently, we expect the flexibility provided by the NIG to improve performance with regards to a traditional Brownian motion approach.

Table 1. Summary Statistics

Energy	Energy-Switching	Oil	Coal	Natural Gas	Fuel-Switching
Mean	61.11	62.05	57.75	4.67	2.55
Median	62.17	59.38	58.02	4.03	2.97
Standard Deviation	16.20	26.89	10.11	2.25	3.98
Min	15.10	17.72	39.50	1.59	-7.16
Max	97.04	145.18	79.50	18.48	14.88
Period	14-18	00-18	10-18	00-18	14-19
Observations	223	992	425	947	1272
Skewness	-0.76	0.36	0.08	1.66	0.050
Kurtosis	0.75	-0.79	-0.47	3.94	2.45

Table 2 describes the result of three energy prices. This paper chose to focus on Energy-switching, oil, and fuel-switching as they are the main focus of this paper. κ represents the speed of mean-reversion of the process. As hinted by the plots in the previous section, we observe that mean-reversion speed of energy-switching is faster than the oil one. Additionally, we note that θ, the average price of the process, is almost equal to the actual mean. Therefore, calibrating the processes using OLS gives satisfactory results. Regarding the Heston model, parameter estimates are not presented in this section since they are hardly comparable with the other two processes, however, the simulation section presents an extensive explanation of the model.

Table 2. OLS Parameter Estimates

Parameter	Energy-Switching	Oil	Fuel-Switching
κ	0.1095921	0.007238064	0.008600791
θ	63.45765	64.84073	1.10466
σ	7.408024	3.064373	0.4007405

Table 3. NIG Parameter Estimates

Parameter	Energy-Switching	Oil	Fuel-Switching
α	1.393878	1.099928	0.60614414
β	0.1155199	-0.1857707	-0.13472828
δ	1.059834	0.9967635	0.10518010
μ	-0.3104575	0.1708004	0.03343841

5.2 Simulation Results

Prices were simulated using the software R and various packages from the CRAN library. As an example, we choose to present the result for energy-switching (main focus) and oil (larger dataset available). As one can see in the plots that follow, the Lévy NIG process incorporates high-spike and larger volatility when compared to the Normal. If it appears clear that the NIG is better to simulate the energy-switching price, the results for oil are harder to assess. Moreover, the Heston model does not seem to fit our data correctly. A possible explanation is that the weekly volatility is not distributed as a χ^2. Therefore, the next part of this section reviews the goodness of fit test for the Normal and NIG processes, in order to confirm our visual interpretation.

5.3 Goodness of Fit Test

The Kolmogorov-Smirnov test is a common approach to estimate the goodness of fit of the data. A p-value larger than five percent indicates that the specified model and the empirical data come from the same distribution. Table 4 gives the p-value for the selected energy prices. The p-values of the Kolmogorov-Smirnov test were calculated based on the parameter estimated and the distribution of the residuals. According to this method, the NIG performs better than the Normal for almost every energy prices except for natural gas. However, these results must be interpreted with caution. Indeed, the

Fig. 5. Simulation Results for Energy-Switching

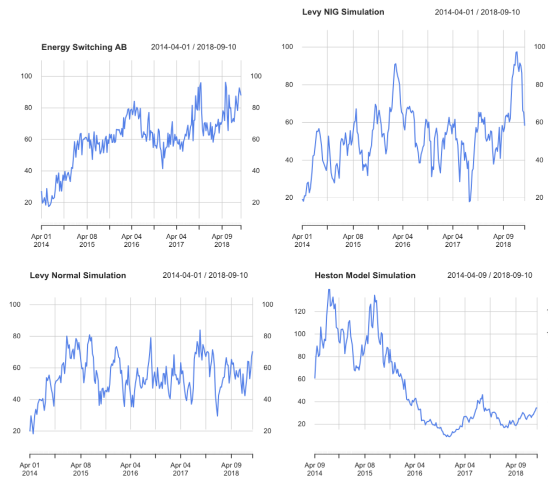
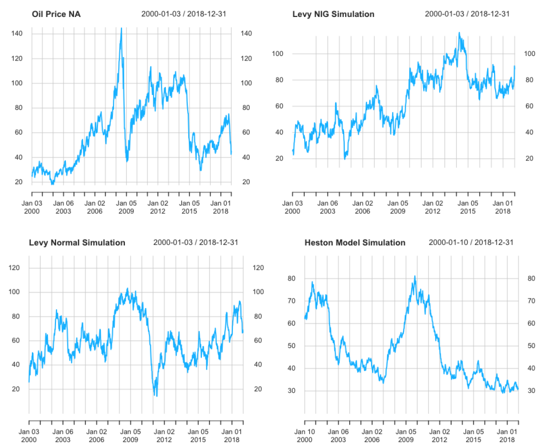


Fig. 6. Simulation Results for Oil



p-values appear to be quite high. Inflated p-values are not uncommon since the parameters were estimated directly using the residuals. Nonetheless, the Kolmogorov-Smirnov test remains the best approach to assess the goodness of fit of the data. We address this issue in greater details in the shortcomings.

Table 4. Kolmogorov-Smirnov Test

P-value	Energy-Switching	Oil	Fuel-Switching	Coal	Natural Gas
Normal	0.3677	0.8609	0.3868	0.704	0.9136
Normal Inverse Gaussian	0.4484	0.869	0.7987	0.762	0.3277

6 Conclusion

6.1 Discussion

This section proposes a discussion of the methodology used and the topics, which should drive further research.

First, the shortcomings of the models and solutions are presented. Second, alternatives to model stochastic pricing are presented. Finally, the results found are compared with the one in the literature.

As mentioned in the previous section, the goodness of fit test of our data suffers from inflated p-values resulting from the estimation technique of our parameter. Another common approach to assess the goodness of fit the Cramer-von Mises criterion. The idea is to compare whether the empirical distribution with the assumed distribution. The value of the test is calculated under a minimum distance estimation procedure (Anderson, 1962).

Additionally, we discovered that the choice of the risk-free rate impacted greatly the outcome of the Heston model simulation. Since energy prices depend a lot on international political decisions, it might be wrong to use the average of a 3-month t-bill interest rate issued by the federal government of the United States. Consequently, we chose to set the risk-free as zero. Further research could focus on the correct estimation of the risk-free in a world where countries have proven to default and where some countries, such as Switzerland and Denmark experience even negative interest rate on their government bonds.

Two alternatives could have been used to model energy prices. First, let us consider the case of a Hawkes process. a Hawkes process is a counting process, as opposed to a Poisson process, possesses a random intensity function. The random intensity parameter increases each time an event takes place, in other words, it relies on the past history of jumps. Therefore, a Hawkes process is said to be a self-exciting process (Bacry et al, 2016). Hence, the idea behind the Hawkes process is to assume it is endogenous. Second, we can also consider the Markov Switching Lévy-driven Ornstein-Uhlenbeck processes, which the parameter σ can be changed under different states of a Markov chain and different state represents different economic status like inflation or a crisis. Hence, future research should try and implement these two types of processes.

The findings of this paper are similar to the one of Goutte and Chevalier (2015), who investigated the fuel-switching price on the European market from 2007 to 2010. At that time, the price of coal was relatively cheap in comparison to the price of natural gas. Their main results indicate that Lévy NIG outperforms the Normal by far. Moreover, they considered the case of Poisson process and showed it was not suitable to model energy prices.

6.2 Summary of Main Results

The goal of this paper is to assess to review the introduction of the carbon tax and to model energy prices. The procedures employed to define the theoretical carbon price are energy-switching and fuel-switching. Furthermore, this article focused on oil, coal and natural gas separately. Three stochastic processes are considered to model prices: Lévy NIG, Lévy Normal and Heston model.

The recent changes in the coal and natural gas prices have made fuel-switching an obsolete method to price carbon. Indeed, the fuel-switching price appears to be negative for several periods. Energy-switching, on the other hand, relies on the use of renewable energies, such as wind. This approach is hard to implement due to the lack of existing infrastructure, which does not allow to switch from one energy to the other. Nonetheless, as the use of renewable increases, a policy maker could define a carbon price based on this approach. Future research could consider alternative sources of energies, such as hydro and solar energies.

The three types of stochastic procedures yield different results. Overall, the Lévy NIG outperforms both, the Lévy Normal and Heston model. Moreover, it seems that the Heston model is not suitable for energy prices. Future research should focus on Hawkes processes and Markov-switching models. Additionally, different frequencies could be used such as daily or even high-frequency data. We chose weekly due to the difficulties to switch from one energy to another on a daily basis.

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