Revenue Management: Models and Algorithms

Huseyin Topaloglu
School of Operations Research and Information Engineering
Cornell University

Sumit Kunnumkal
Indian School of Business

Revenue Management

- Making most use of **limited** inventories of **perishable** resources under **random demand**
- Airlines form a typical application setting
  - Resources correspond to capacities on flight legs
  - Customers arrive randomly over time
- Other application settings include hotels, car rentals, advertising, fashion retail
- Primary tradeoff is whether to give resources to a currently available customer that is willing to pay a low price or keep the resources with the hope of a future customer that may be willing to pay a high price

Revenue Management

- Different companies use different decision making mechanisms in their revenue management activities
- Customers may arrive with specific willingness to pay and the revenue management decision may involve **accepting or rejecting** the customer
- Companies may **adjust the prices** as a function of the remaining resource capacity and remaining time in the selling horizon
- Companies may make different **assortments of products** available to their customers and customers make a choice within the offered assortment

Revenue Management

- Different companies use different decision making mechanisms in their revenue management activities
- **Capacity allocation**
- Companies may **adjust the prices** as a function of the remaining resource capacity and remaining time in the selling horizon
- Companies may make different **assortments of products** available to their customers and customers make a choice within the offered assortment

Revenue Management

- Different companies use different decision making mechanisms in their revenue management activities
- **Capacity allocation**
- **Dynamic pricing**
- Companies may make different **assortments of products** available to their customers and customers make a choice within the offered assortment

Revenue Management

- Different companies use different decision making mechanisms in their revenue management activities
- **Capacity allocation**
- **Dynamic pricing**
- **Assortment planning**

Revenue Management

- Capacity allocation over single flight leg
- Capacity allocation over an airline network

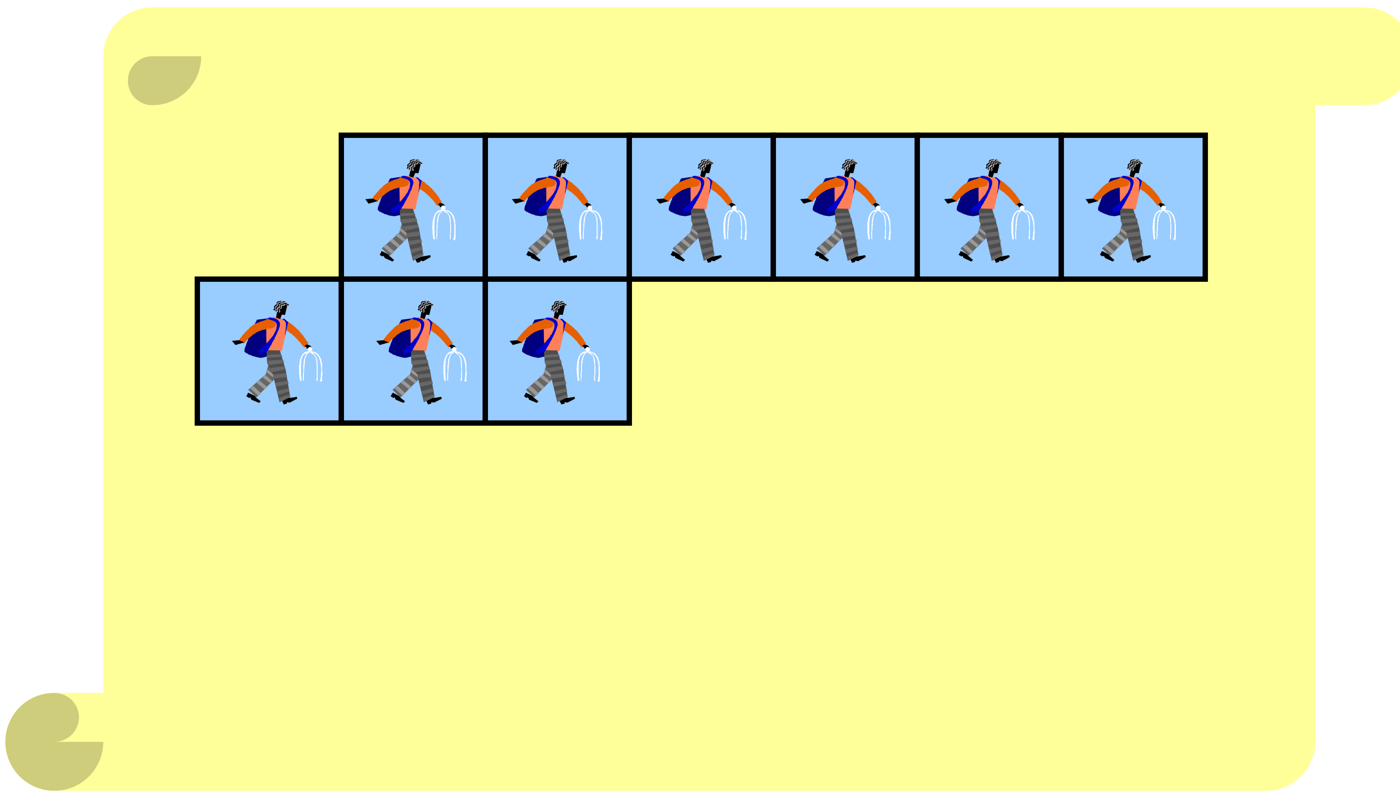
Capacity Allocation Over Single Flight Leg

Capacity Allocation Over Single Flight Leg

- We have n fare classes indexed by j
- Demand from each fare class is observed sequentially

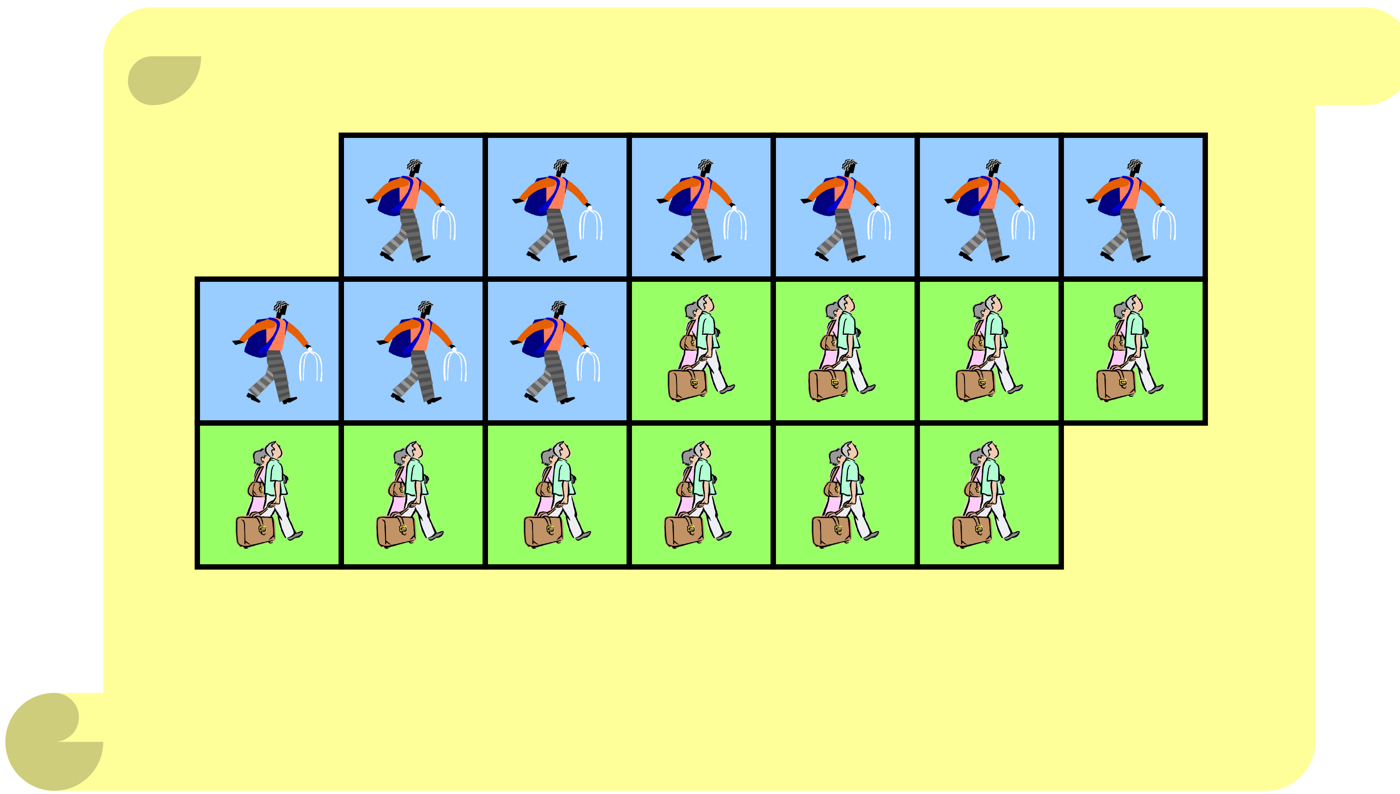
Capacity Allocation Over Single Flight Leg

- We have n fare classes indexed by j
- Demand from each fare class is observed sequentially



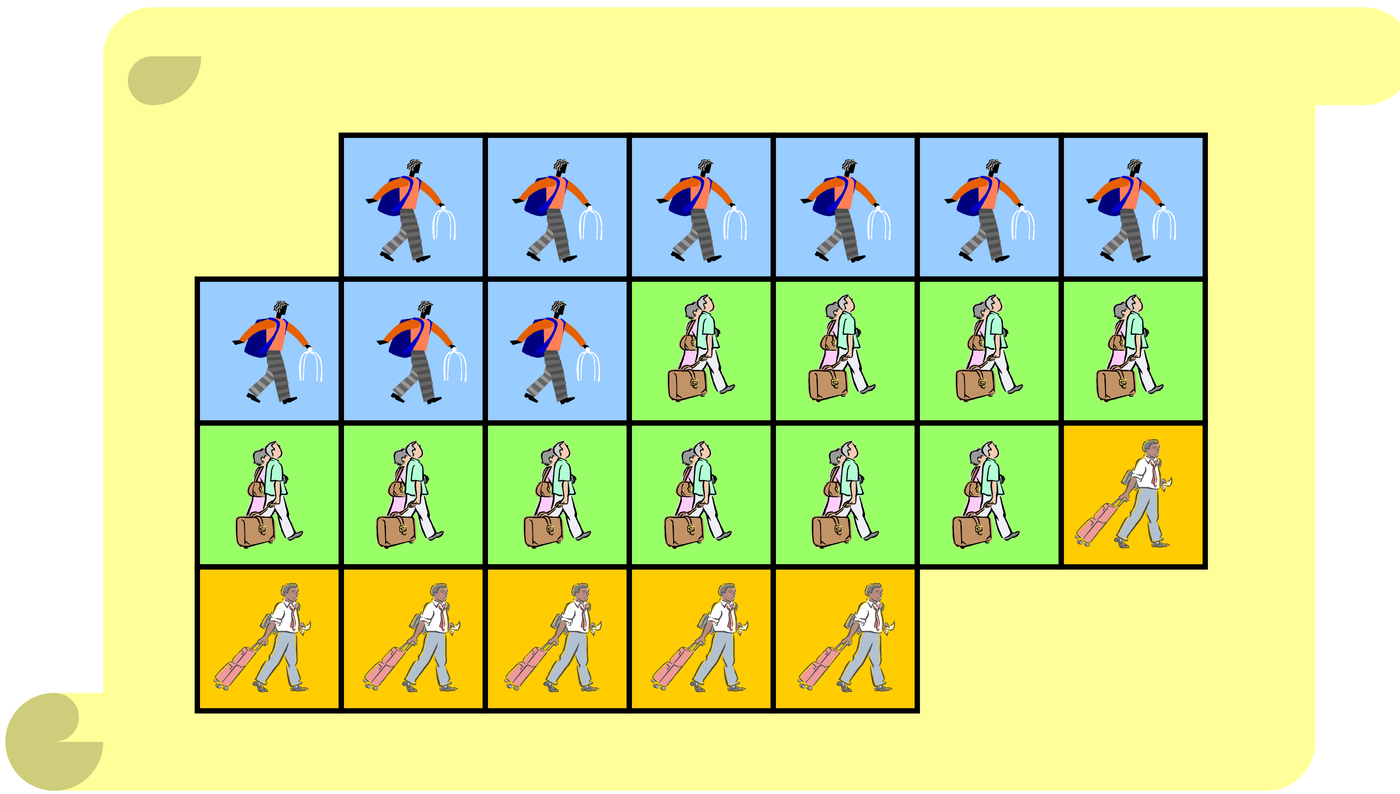
Capacity Allocation Over Single Flight Leg

- We have n fare classes indexed by j
- Demand from each fare class is observed sequentially



Capacity Allocation Over Single Flight Leg

- We have n fare classes indexed by j
- Demand from each fare class is observed sequentially



Capacity Allocation Over Single Flight Leg

- D_j : Demand from fare class j
- f_j : Revenue from fare class j
- x_j : Capacity left just before making the decisions for fare class j
- y_j : Capacity left just after making the decisions for fare class j

$$V_j(x_j) = \mathbb{E}\left\{ \max_{x_j - D_j \le y_j \le x_j} \left\{ f_j \,(x_j - y_j) + V_{j+1}(y_j) \right\} \right\}$$

Capacity Allocation Over Single Flight Leg

- D_j : Demand from fare class j
- f_j : Revenue from fare class j
- x_j : Capacity left just before making the decisions for fare class j
- y_j : Capacity left just after making the decisions for fare class j

$$V_j(x_j) = \mathbb{E}\left\{ \max_{[x_j - D_j]^+ \le y_j \le x_j} \left\{ f_j\,(x_j - y_j) + V_{j+1}(y_j) \right\} \right\}$$

Capacity Allocation Over Single Flight Leg

- D_j : Demand from fare class j
- f_j : Revenue from fare class j
- x_j : Capacity left just before making the decisions for fare class j
- y_j : Capacity left just after making the decisions for fare class j

$$V_j(x_j) = \mathbb{E}\left\{ \max_{[x_j - D_j]^+ \le y_j \le x_j} \left\{ f_j\,(x_j - y_j) + V_{j+1}(y_j) \right\} \right\}$$

- If the capacity left just before making the decision for fare class j is $\boldsymbol{x_j}$ and the demand from this fare class is $\boldsymbol{D_j}$, then the optimal decision is obtained by solving

$$\max_{[x_j - D_j]^+ \le y_j \le x_j} \left\{ f_j\,(x_j - y_j) + V_{j+1}(y_j) \right\}$$

Capacity Allocation Over Single Flight Leg

- D_j : Demand from fare class j
- f_j : Revenue from fare class j
- x_j : Capacity left just before making the decisions for fare class j
- y_j : Capacity left just after making the decisions for fare class j

$$V_j(x_j) = \mathbb{E}\left\{ \max_{[x_j - D_j]^+ \le y_j \le x_j} \left\{ f_j \, (x_j - y_j) + V_{j+1}(y_j) \right\} \right\}$$

- If the capacity left just before making the decision for fare class j is x_j and the demand from this fare class is D_j, then the optimal decision is obtained by solving

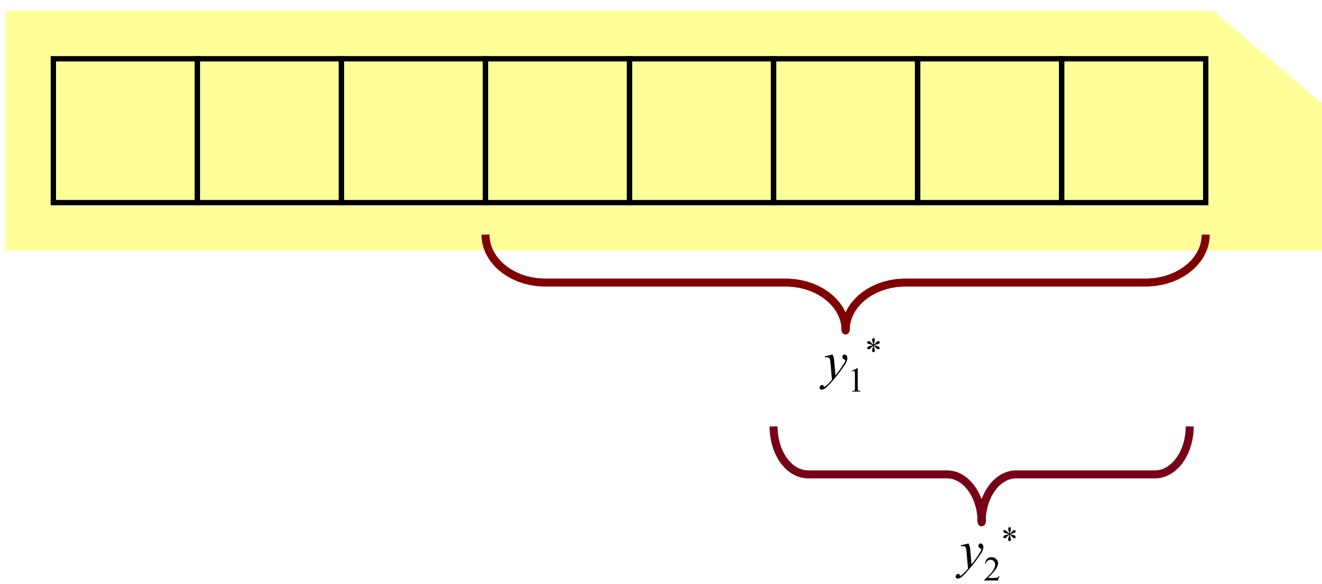
$$\max_{[x_j - D_j]^+ \le y_j \le x_j} \left\{ - f_j \, y_j + V_{j+1}(y_j) \right\}$$

Protection Levels

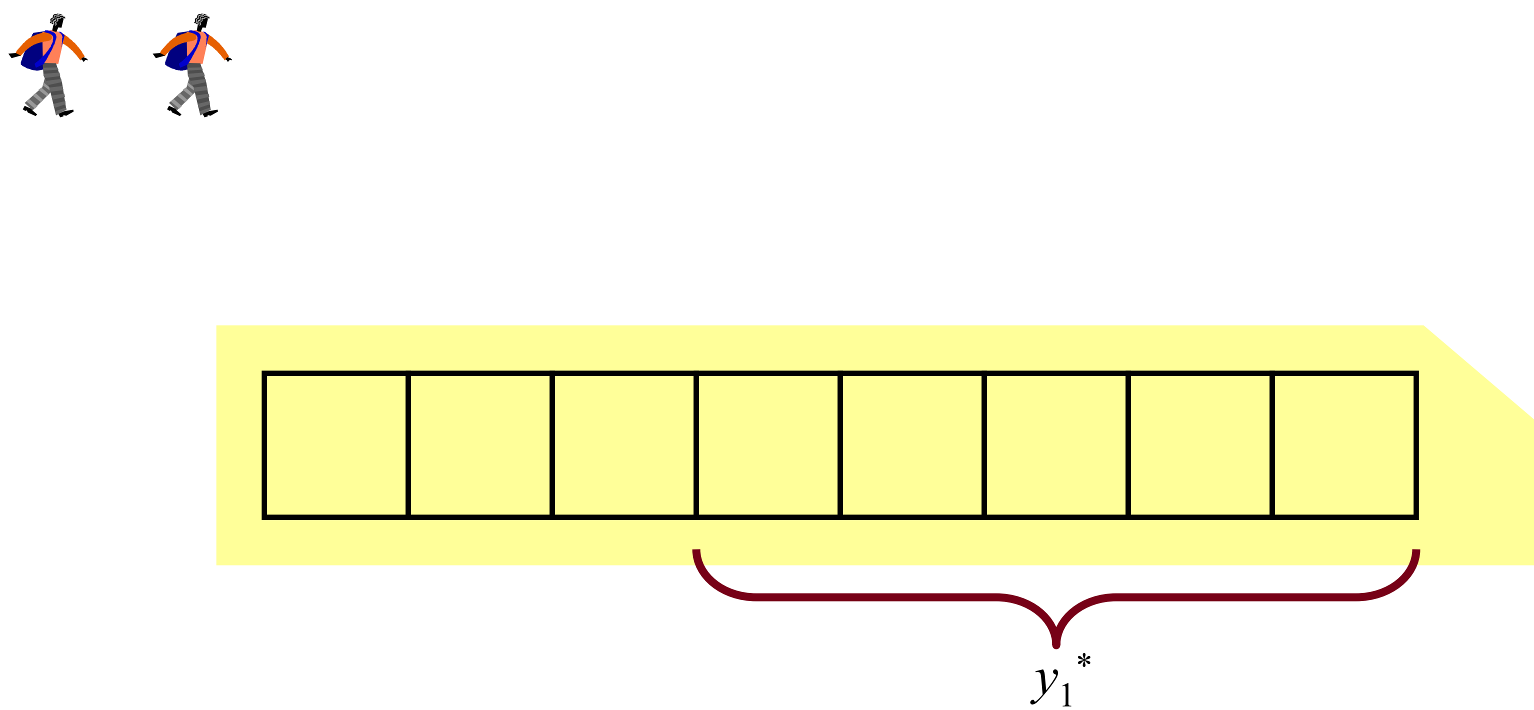
- The optimal policy is characterized by n protection levels, which we denote by $y_1^*, y_2^*, \ldots, y_n^*$
- When making the decision for fare class j, it is optimal to protect y_j^* seats for the later fare classes

Protection Levels

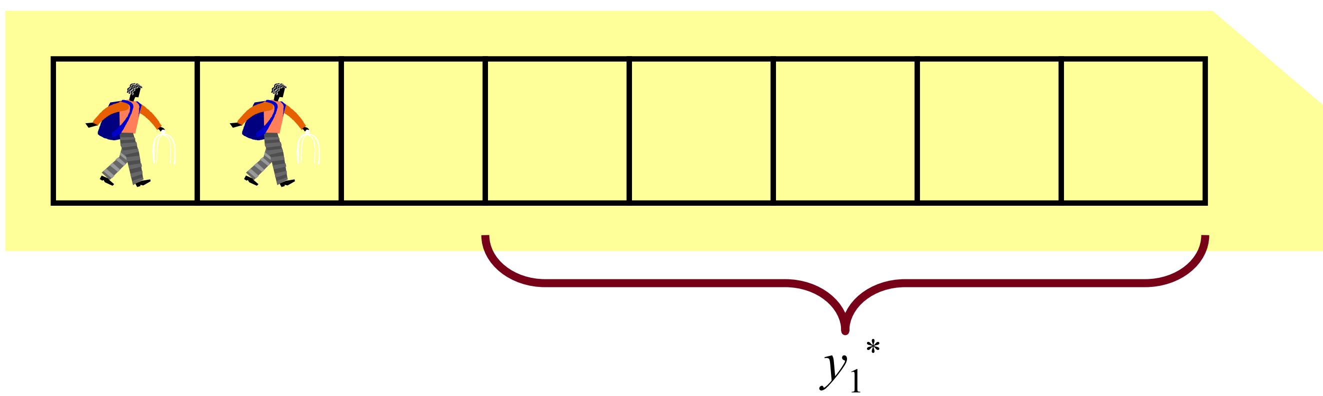
- The optimal policy is characterized by n protection levels, which we denote by $y_1^*, y_2^*, \ldots, y_n^*$
- When making the decision for fare class j, it is optimal to protect y_j^* seats for the later fare classes



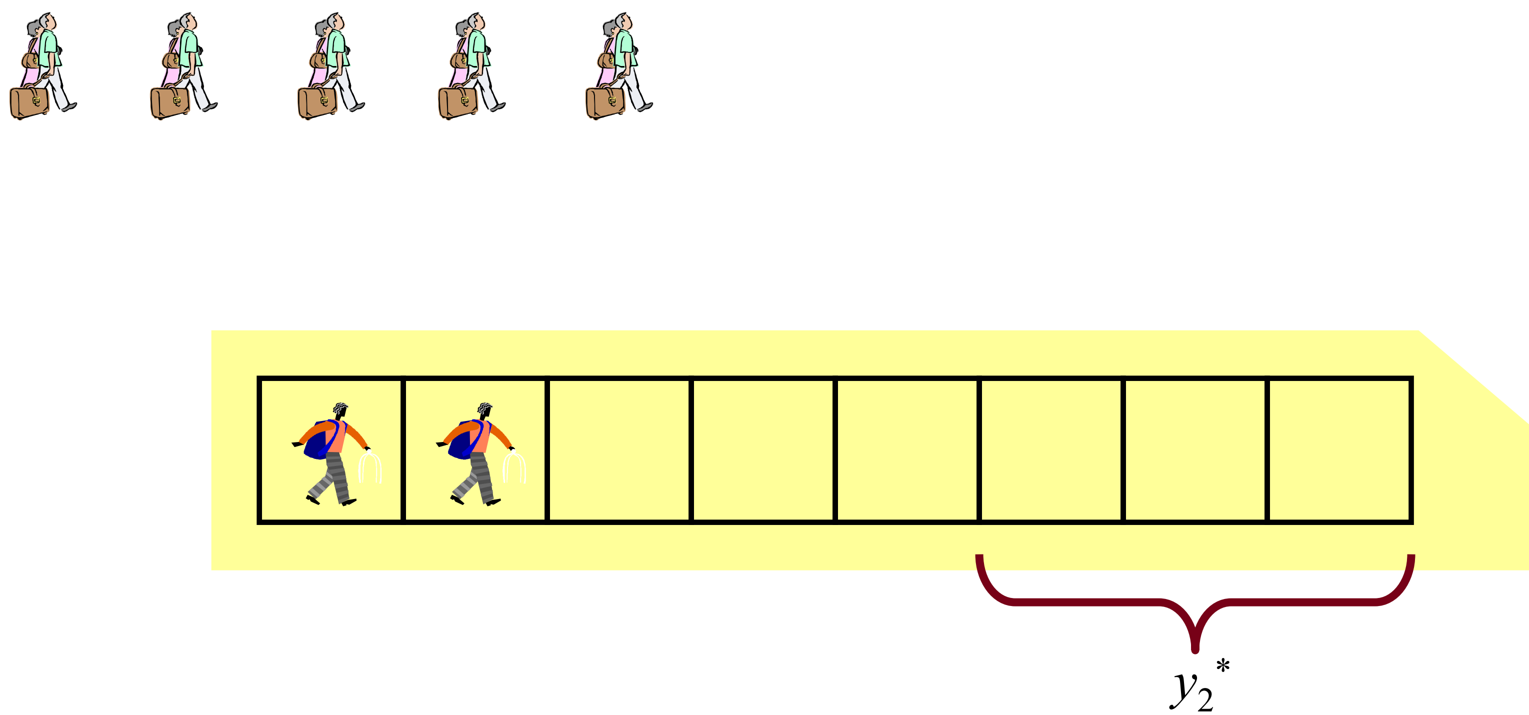
Protection Levels



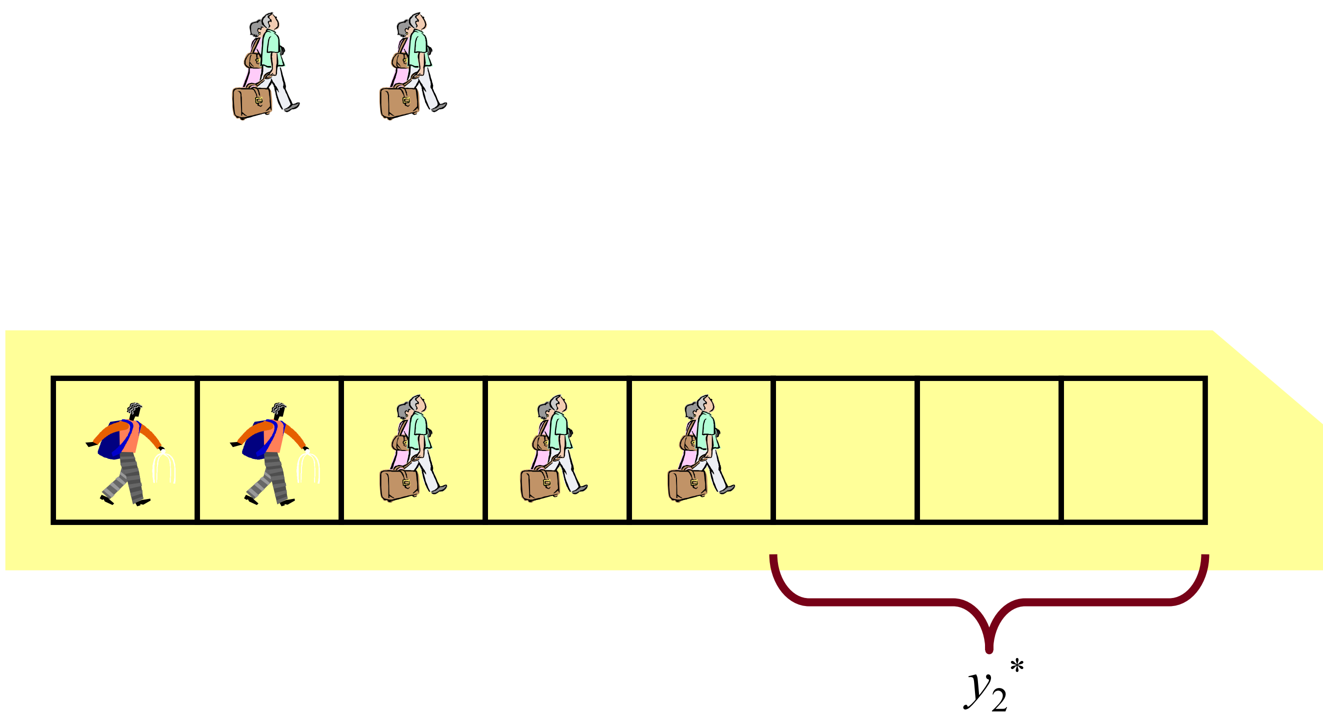
Protection Levels



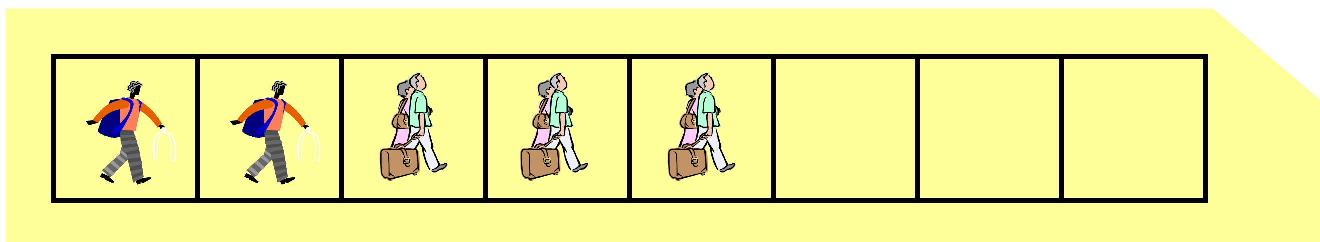
Protection Levels



Protection Levels



Protection Levels



Protection Levels

Structure of the Optimal Policy

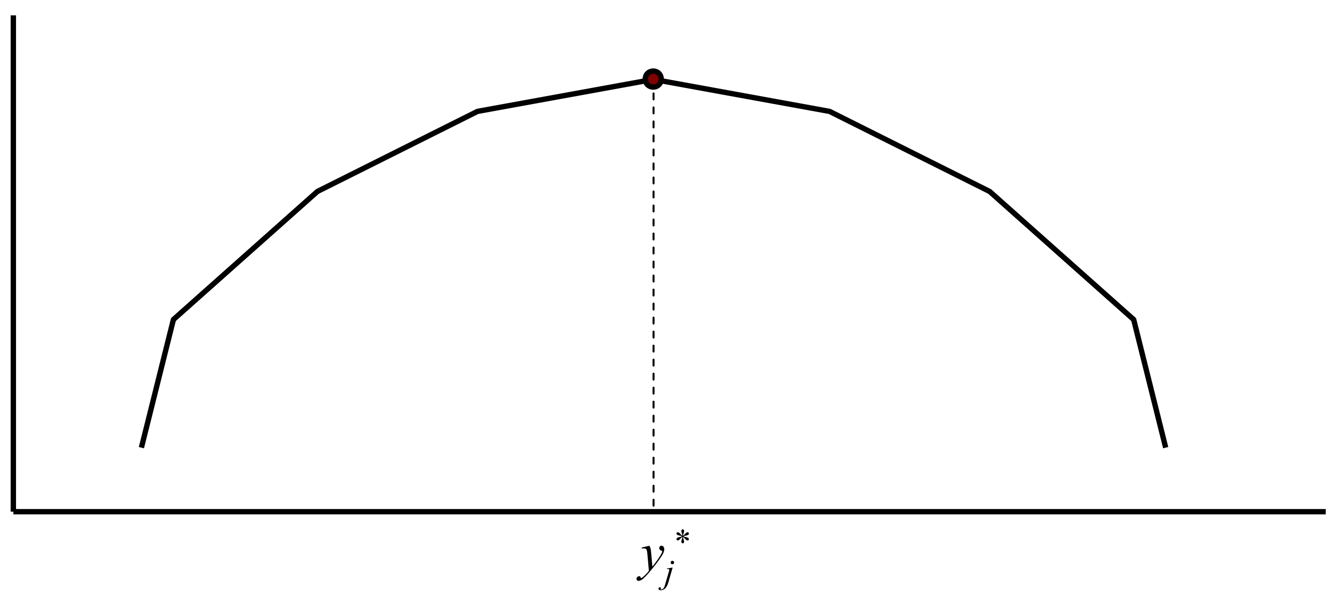
- y_j^* is the unconstrained maximizer of $-f_j\, y_j + V_{j+1}(y_j)$
- If the capacity left just before making the decision for fare class j is $\boldsymbol{x_j}$ and the demand from this fare class is $\boldsymbol{D_j}$, then the optimal decision is obtained by solving

$$\max_{[x_j - D_j]^+ \le y_j \le x_j} \Big\{ -f_j\, y_j + V_{j+1}(y_j) \Big\}$$

Structure of the Optimal Policy

- y_j^* is the unconstrained maximizer of $-f_j\, y_j + V_{j+1}(y_j)$
- If the capacity left just before making the decision for fare class j is x_j and the demand from this fare class is D_j, then the optimal decision is obtained by solving

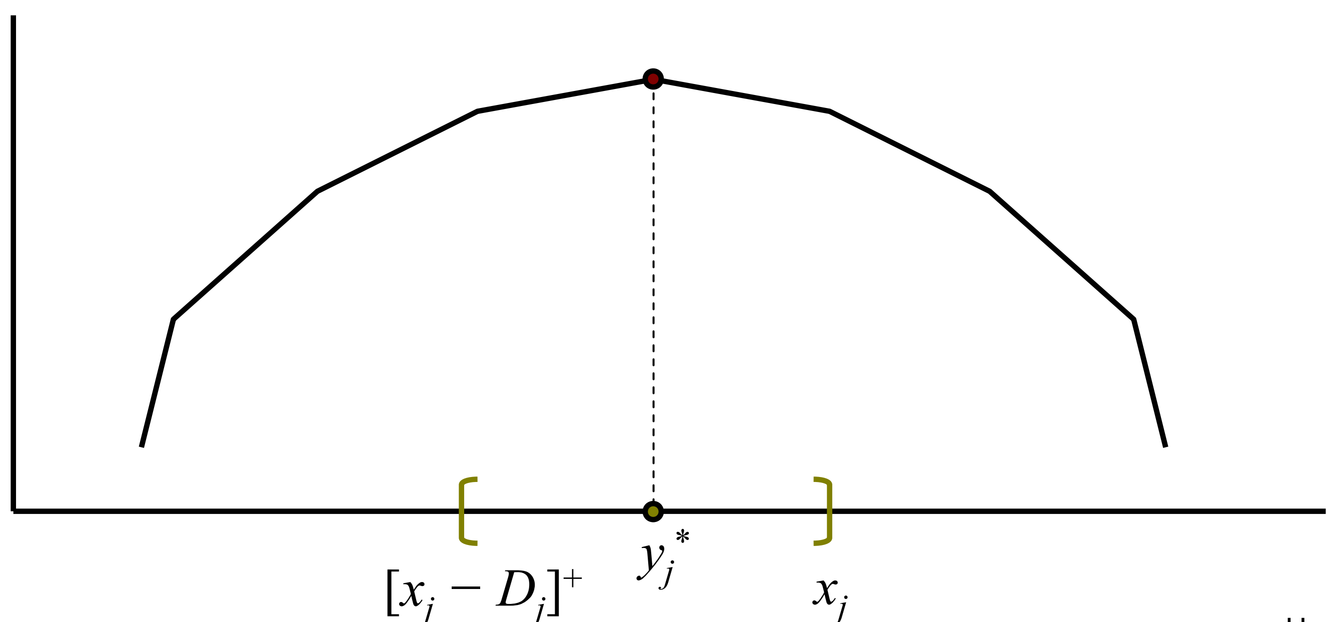
$$\max_{[x_j - D_j]^+ \le y_j \le x_j} \Big\{ -f_j\, y_j + V_{j+1}(y_j) \Big\}$$



Structure of the Optimal Policy

- y_j^* is the unconstrained maximizer of $-f_j\, y_j + V_{j+1}(y_j)$
- If the capacity left just before making the decision for fare class j is $\boldsymbol{x_j}$ and the demand from this fare class is $\boldsymbol{D_j}$, then the optimal decision is obtained by solving

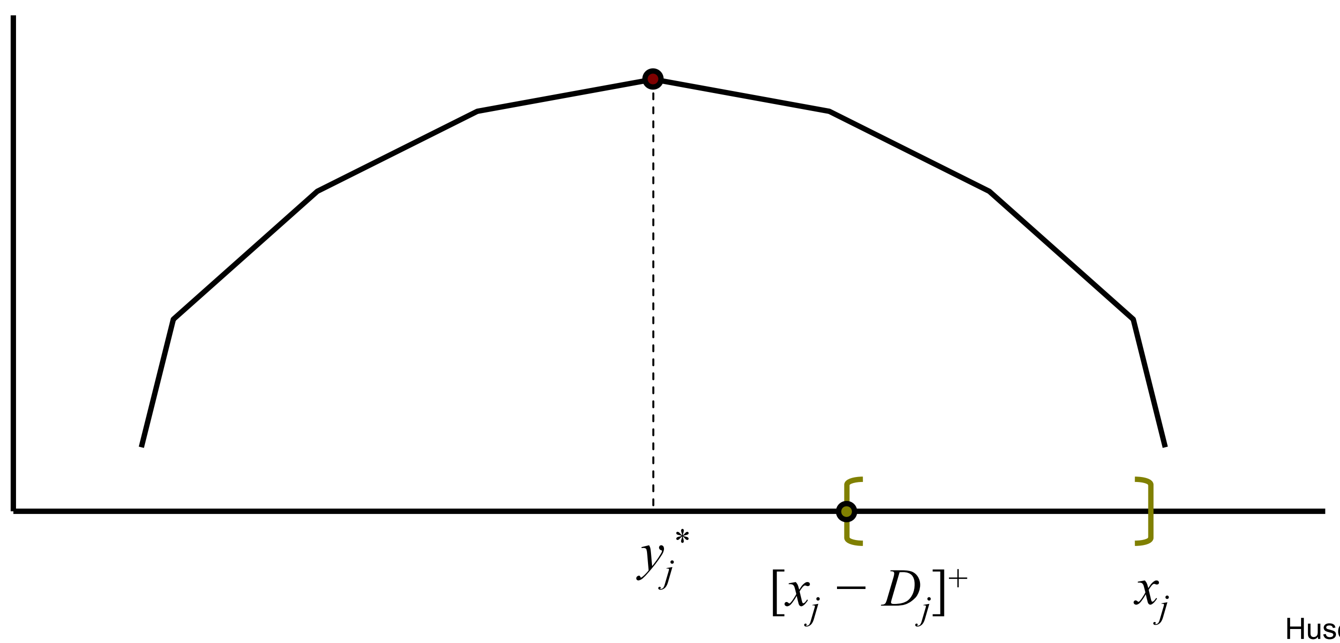
$$\max_{[x_j - D_j]^+ \leq y_j \leq x_j} \left\{ - f_j\, y_j + V_{j+1}(y_j) \right\}$$



Structure of the Optimal Policy

- y_j^* is the unconstrained maximizer of $-f_j\, y_j + V_{j+1}(y_j)$
- If the capacity left just before making the decision for fare class j is x_j and the demand from this fare class is D_j, then the optimal decision is obtained by solving

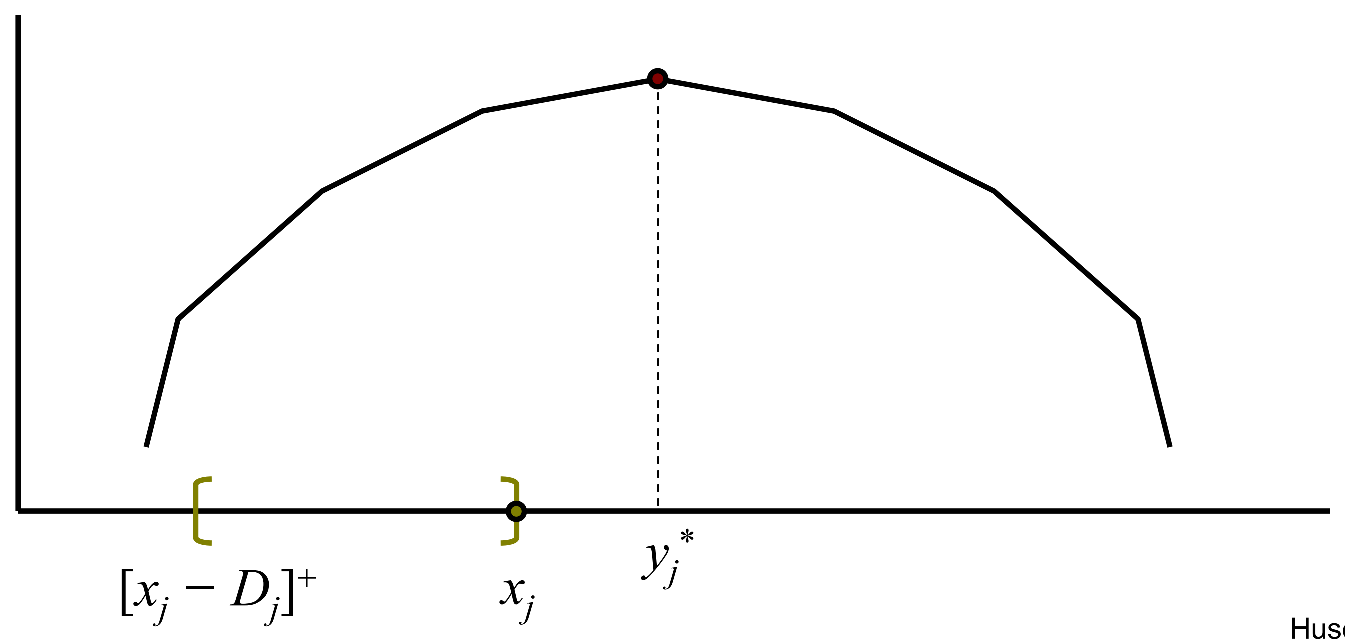
$$\max_{[x_j - D_j]^+ \le y_j \le x_j} \left\{ -f_j\, y_j + V_{j+1}(y_j) \right\}$$



Structure of the Optimal Policy

- y_j^* is the unconstrained maximizer of $-f_j\, y_j + V_{j+1}(y_j)$
- If the capacity left just before making the decision for fare class j is x_j and the demand from this fare class is D_j, then the optimal decision is obtained by solving

$$\max_{[x_j - D_j]^+ \leq y_j \leq x_j} \Big\{ -f_j\, y_j + V_{j+1}(y_j) \Big\}$$



Structure of the Value Functions

- Value of an additional unit of capacity decreases as more capacity is available

$$V_j(x+1) - V_j(x) \leq V_j(x) - V_j(x-1)$$

- Capacity that becomes available earlier is more valuable

$$V_{j+1}(x+1) - V_{j+1}(x) \leq V_j(x+1) - V_j(x)$$

Structure of the Value Functions

- Value of an additional unit of capacity decreases as more capacity is available

$$V_j(x+1) - V_j(x) \leq V_j(x) - V_j(x-1)$$

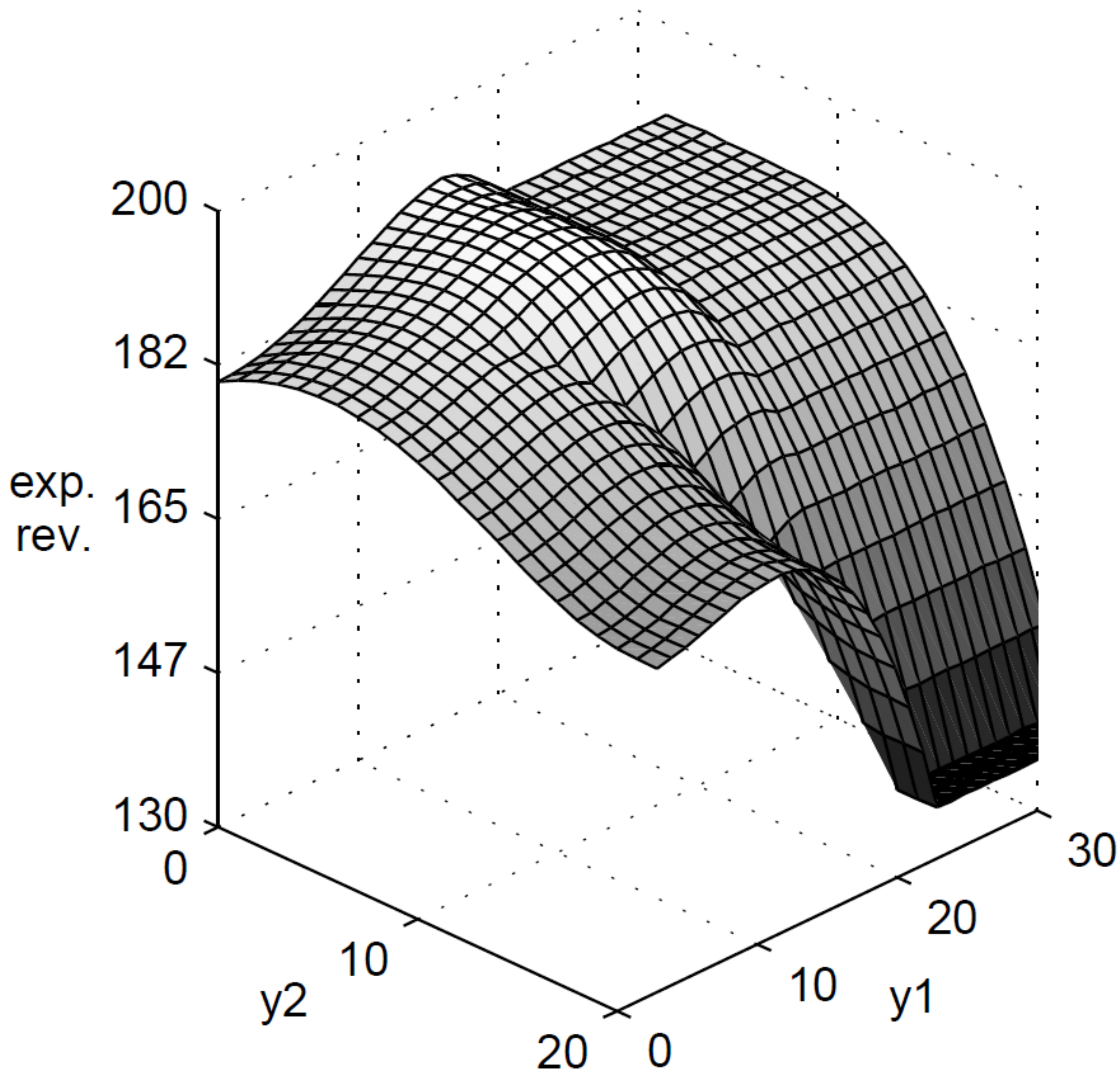
- Capacity that becomes available earlier is more valuable

$$V_{j+1}(x+1) - V_{j+1}(x) \leq V_j(x+1) - V_j(x)$$

- Protection levels are nested satisfying $y_1^* \geq y_2^* \geq \ldots \geq y_n^*$

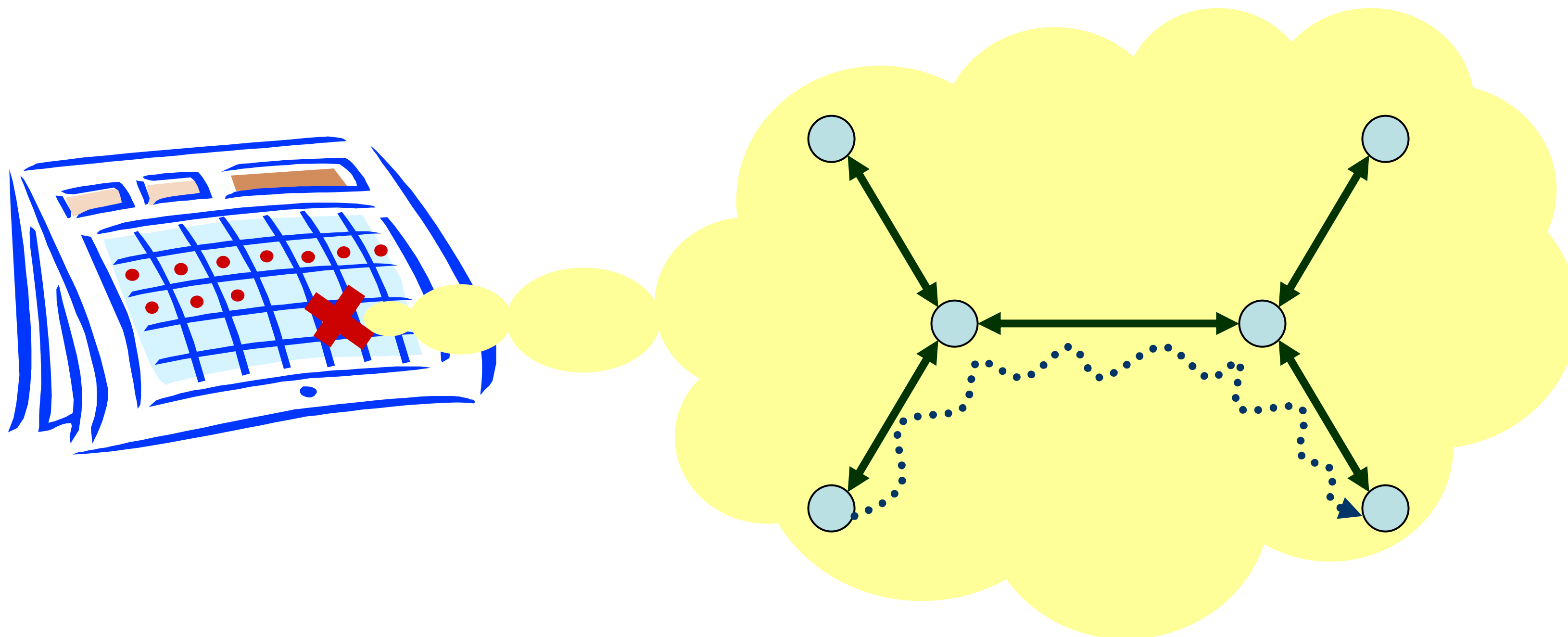
Direct Optimization Over Protection Levels

- Direct optimization over protection levels is difficult



Capacity Allocation Over an Airline Network

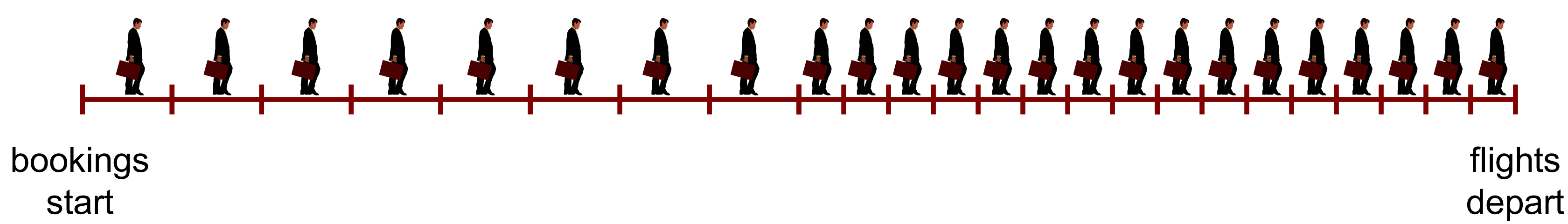
Network Revenue Management



- Requests arrive over time
- Make an acceptance or rejection decision for each request
- Accepted requests consume capacities on one or more flight legs

A Deterministic Linear Programming Approximation

- $\mathcal{L}$: Set of flight legs in the airline network
- $\mathcal{J}$: Set of itineraries
- p_{jt} : Probability of getting a request for itinerary j at time t
- f_j : Revenue from itinerary j
- a_{ij} : 1 if itinerary j uses flight leg i
  0 otherwise
- c_i : Total available capacity on flight leg i



A Deterministic Linear Programming Approximation

- Assume that the numbers of itinerary requests take on their expected values

$$
\begin{aligned}
LP = \max \quad & \sum_{j \in \mathcal{J}} f_j \, w_j \\
\text{subject to} \quad & \sum_{j \in \mathcal{J}} a_{ij} \, w_j \leq c_i \qquad i \in \mathcal{L} \quad (\mu_i) \\
& 0 \leq w_j \leq \sum_{t \in \mathcal{T}} p_{jt} \qquad j \in \mathcal{J}
\end{aligned}
$$

A Deterministic Linear Programming Approximation

- $\{\mu_i : i \in \mathcal{L}\}$ are called bid prices
- They characterize the value of a seat
- Accept a request for itinerary j if

$$f_j \geq \sum_{i \in \mathcal{L}} a_{ij} \, \mu_i$$

...subject to capacity availability

- As time progresses, it is customary to refresh the bid prices by resolving the linear program
- Optimal objective value of the linear program provides an upper bound on the performance of any nonancitipatory policy

Dynamic Programming Formulation

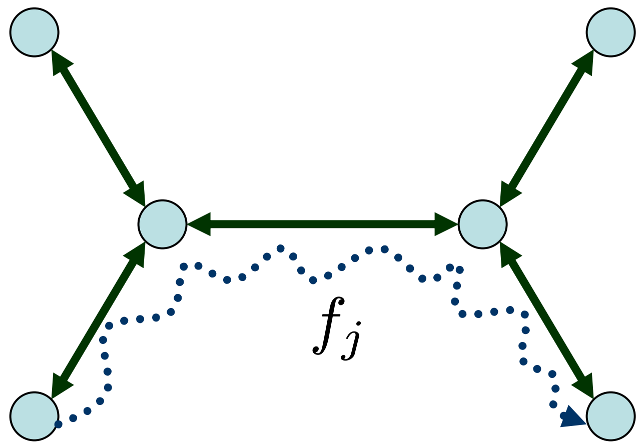
- $\mathcal{L}$: Set of flight legs in the airline network
- $\mathcal{J}$: Set of itineraries
- p_{jt} : Probability of getting a request for itinerary j at time t
- f_j : Revenue from itinerary j
- a_{ij} : 1 if itinerary j uses flight leg i
  0 otherwise
- x_{it} : Remaining capacity on flight leg i at time t
- u_{jt} : 1 if a request for itinerary j is accepted at time t
  0 otherwise

Dynamic Programming Formulation

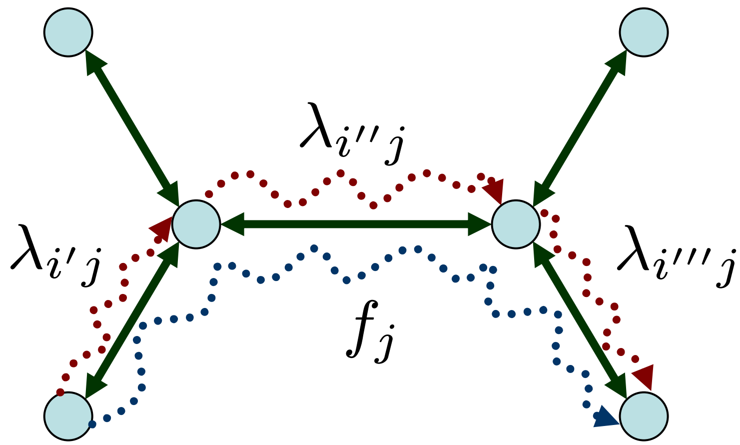
$$\begin{aligned} V_t(x_t) = \max \quad & \sum_{j \in \mathcal{J}} p_{jt} \Big\{ f_j \, u_{jt} + V_{t+1}\big(x_t - u_{jt} \textstyle\sum_{i \in \mathcal{L}} a_{ij} \, e_i\big) \Big\} \\ \text{subject to} \quad & a_{ij} \, u_{jt} \leq x_{it} \qquad i \in \mathcal{L}, \; j \in \mathcal{J} \\ & u_{jt} \in \{0, 1\} \qquad j \in \mathcal{J} \end{aligned}$$

- To obtain approximations to value functions, **decompose** the dynamic programming formulation by the flight legs

Dynamic Programming Decomposition



Dynamic Programming Decomposition



- λ_{ij} : Fare allocation of itinerary j over flight leg i

$$\sum_{i \in \mathcal{L}} \lambda_{ij} = f_j$$

Dynamic Programming Decomposition

- Revenue allocations allow us to solve single-leg problems

$$v_t^i(x_{it} \,|\, \lambda) = \max \quad \sum_{j \in \mathcal{J}} p_{jt} \Big\{ \lambda_{ij}\, u_{jt} + v_{t+1}^i(x_{it} - a_{ij}\, u_{jt} \,|\, \lambda) \Big\}$$

$$\text{subject to} \quad a_{ij}\, u_{jt} \le x_{it} \qquad j \in \mathcal{J}$$

$$u_{jt} \in \{0, 1\} \qquad j \in \mathcal{J}$$

- Single-leg problems provide an upper bound on exact value functions

$$\sum_{i \in \mathcal{L}} v_t^i(x_{it} \,|\, \lambda) \ge V_t(x_t)$$

as long as

$$\sum_{i \in \mathcal{L}} \lambda_{ij} = f_j \quad j \in \mathcal{J}$$

Dynamic Programming Decomposition

- We obtain upper bounds on the optimal expected revenue

$$\sum_{i \in \mathcal{L}} v_t^i(x_{it} \,|\, \lambda) \geq V_t(x_t) \qquad \sum_{i \in \mathcal{L}} v_1^i(c_i \,|\, \lambda) \geq V_1(c)$$

- To obtain the tightest possible upper bound, we solve

$$\begin{aligned} \min_{\lambda} \quad & \sum_{i \in \mathcal{L}} v_1^i(c_i \,|\, \lambda) \\ \text{subject to} \quad & \sum_{i \in \mathcal{L}} \lambda_{ij} = f_j \quad j \in \mathcal{J} \end{aligned}$$

convex in λ

- Tightest possible upper bound is better than the one from deterministic linear programming approximation

$$LP \geq \sum_{i \in \mathcal{L}} v_1^i(c_i \,|\, \lambda^*) \geq V_1(c)$$

Dynamic Programming Decomposition

- We can obtain a good revenue allocation from deterministic linear programming approximation

$$
\begin{aligned}
\max \quad & \sum_{j \in \mathcal{J}} f_j \, w_j \\
\text{subject to} \quad & \sum_{j \in \mathcal{J}} a_{ij} \, w_j \leq c_i && i \in \mathcal{L} \\
& 0 \leq w_j \leq \sum_{t \in \mathcal{T}} p_{jt} && j \in \mathcal{J}
\end{aligned}
$$

Dynamic Programming Decomposition

- We can obtain a good revenue allocation from deterministic linear programming approximation

$$
\begin{aligned}
\max \quad & \sum_{j \in \mathcal{J}} f_j \, w_{\psi j} \\
\text{subject to} \quad & \sum_{j \in \mathcal{J}} a_{ij} \, w_{ij} \le c_i && i \in \mathcal{L} \\
& 0 \le w_{ij} \le \sum_{t \in \mathcal{T}} p_{jt} && i \in \mathcal{L}, \; j \in \mathcal{J} \\
& w_{\psi j} - w_{ij} = 0 && j \in \mathcal{J}, \; i \in \mathcal{L}
\end{aligned}
$$

Dynamic Programming Decomposition

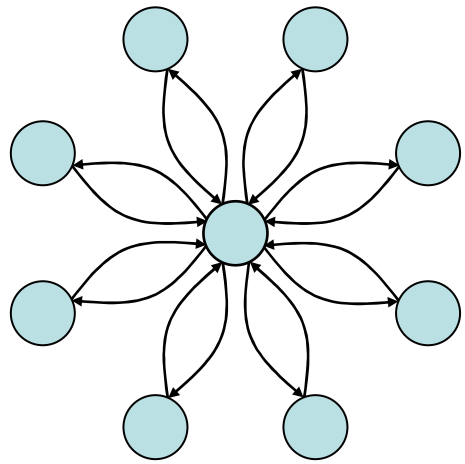
- We can obtain a good revenue allocation from deterministic linear programming approximation

$$
\begin{aligned}
\max \quad & \sum_{j \in \mathcal{J}} f_j \, w_{\psi j} \\
\text{subject to} \quad & \sum_{j \in \mathcal{J}} a_{ij} \, w_{ij} \le c_i && i \in \mathcal{L} \\
& 0 \le w_{ij} \le \sum_{t \in \mathcal{T}} p_{jt} && i \in \mathcal{L}, \; j \in \mathcal{J} \\
& w_{\psi j} - w_{ij} = 0 && j \in \mathcal{J}, \; i \in \mathcal{L} \quad (\lambda_{ij})
\end{aligned}
$$

$$
\sum_{i \in \mathcal{L}} \lambda_{ij} = f_j \qquad j \in \mathcal{J}
$$

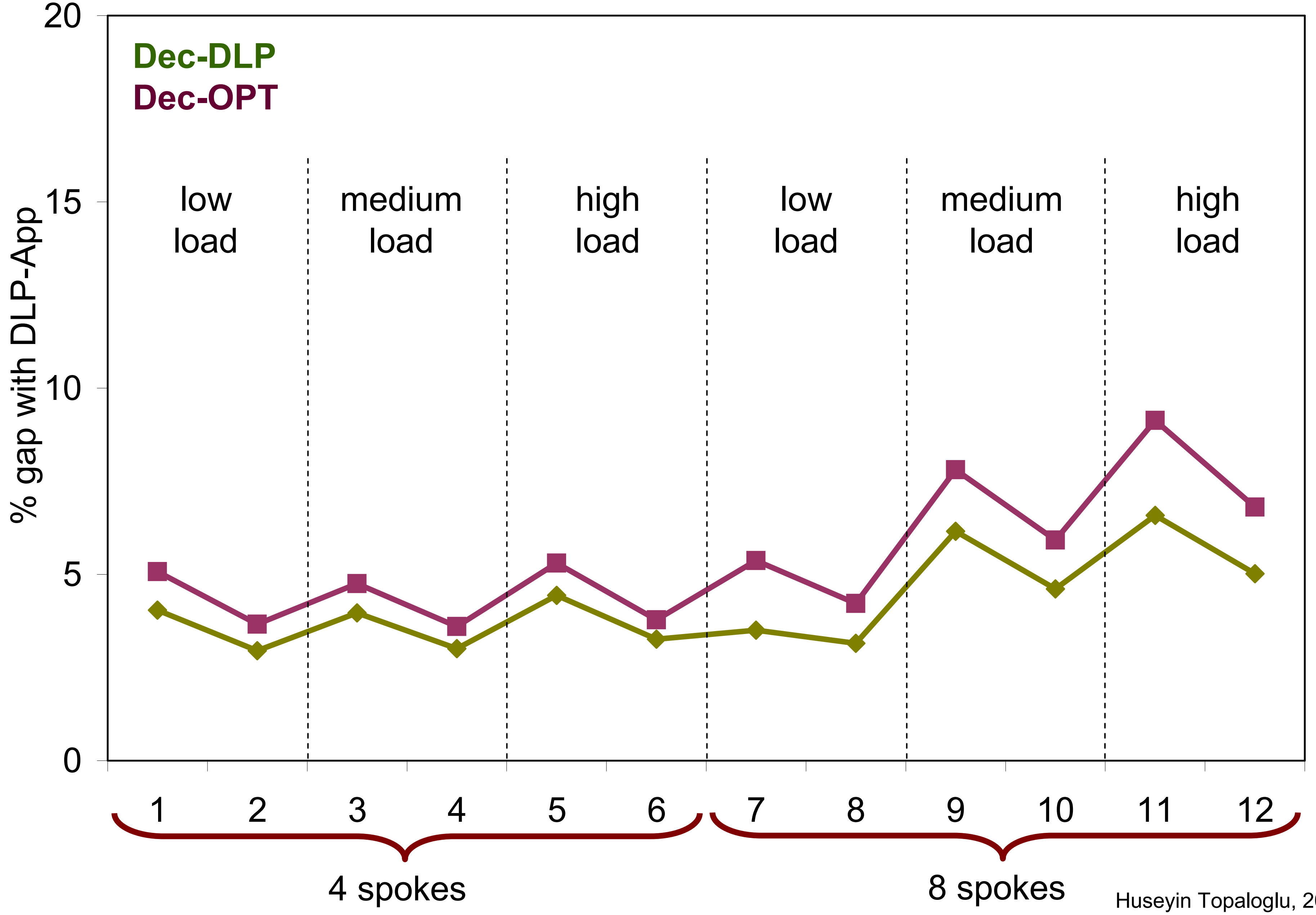
Practical Performance

- An airline network with one hub serving multiple spokes

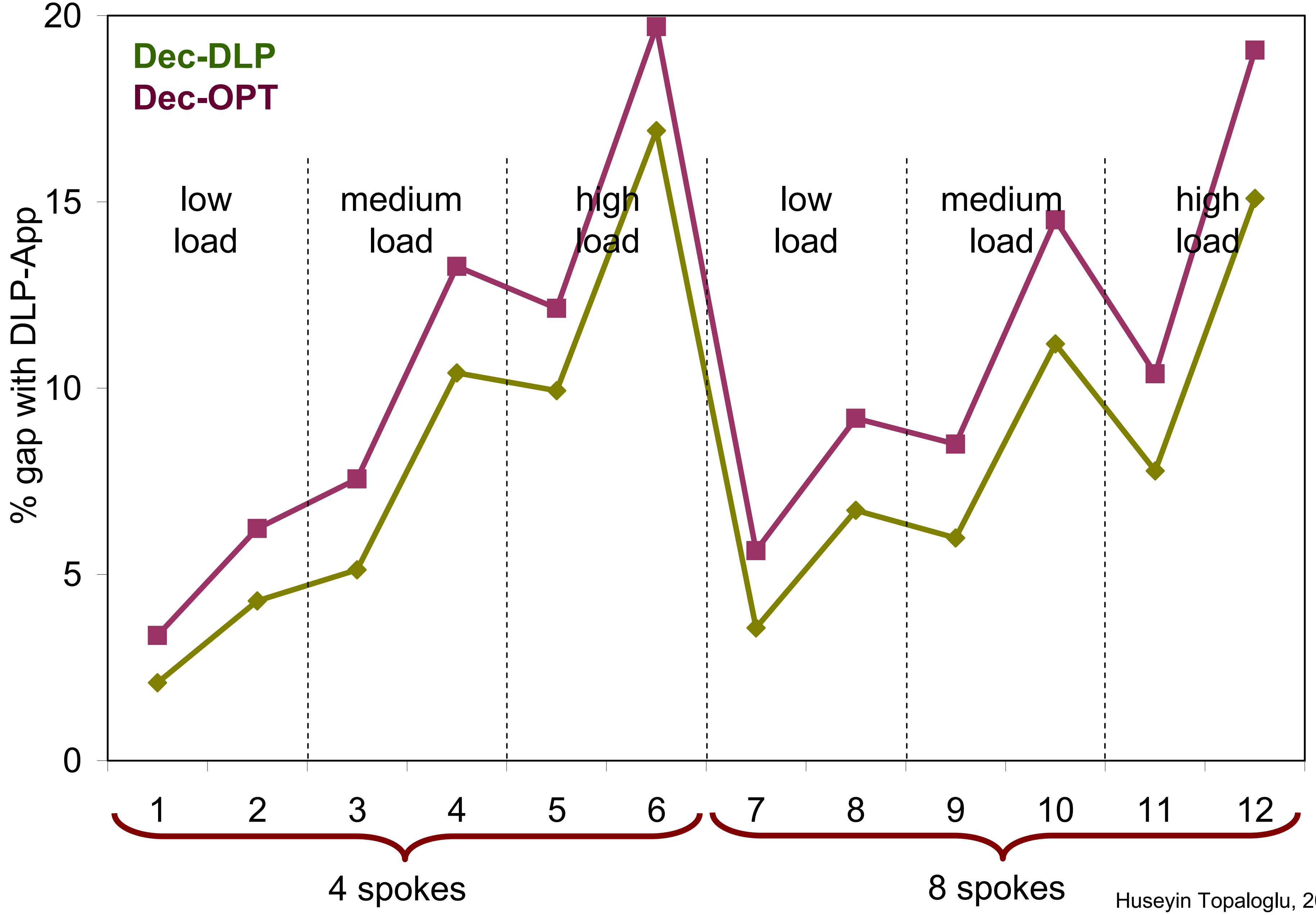


- There is a high-fare and a low-fare itinerary connecting each origin-destination pair
- Compare **deterministic linear programming approximation (DLP-App)**, **decomposition with revenue allocations from the deterministic linear program (Dec-DLP)** and **decomposition with optimal revenue allocations (Dec-OPT)**

Comparing Upper Bounds



Comparing Expected Revenues



(*Very Incomplete*) List of References

- Talluri, K. T. and van Ryzin, G. J. (2004), The Theory and Practice of Revenue Management, Kluver Academic Publishers. (**Book**)
- Phillips, R. L. (2005), Pricing and Revenue Optimization, Stanford University Press, Stanford, CA. (**Book**)

- Simpson, R. W. (1989), Using network flow techniques to find shadow prices for market and seat inventory control, Technical report, MIT Flight Transportation Laboratory Memorandum M89-1, Cambridge, MA. (**Network, capacity allocation**)
- Williamson, E. L. (1992), Airline Network Seat Control, PhD thesis, Massachusetts Institute of Technology, Cambridge, MA. (**Network, capacity allocation**)
- Brumelle, S. L. and McGill, J. I. (1993), "Airline seat allocation with multiple nested fare classes", Operations Research 41, 127-137. (**Single-leg, capacity allocation**)
- Gallego, G. and van Ryzin, G. (1994), “Optimal dynamic pricing of inventories with stochastic demand over finite horizons", *Management Science* 40(8), 999-1020. (**Single-leg, dynamic pricing**)
- Gallego, G. and van Ryzin, G. (1997), “A multiproduct dynamic pricing problem and its applications to yield management”, *Operations Research* 45(1), 24-41. (**Network, dynamic pricing**)
- Talluri, K. and van Ryzin, G. (1998), “An analysis of bid-price controls for network revenue management”, Management Science 44(11), 1577-1593. (**Network, capacity allocation**)
- Talluri, K. and van Ryzin, G. (1999), “A randomized linear programming method for computing network bid prices”, Transportation Science 33(2), 207-216. (**Network, capacity allocation**)
- Bertsimas, D. and Popescu, I. (2003), “Revenue management in a dynamic network environment”, Transportation Science 37, 257-277. (**Network, capacity allocation, overbooking**)
- Talluri, K. and van Ryzin, G. (2004), "Revenue management under a general discrete choice model of consumer behavior", *Management Science* 50(1), 15-33. (**Single-leg, assortment planning**)

(*Very Incomplete*) List of References

- Gallego, G., Iyengar, G., Phillips, R. and Dubey, A. (2004), Managing flexible products on a network, Computational Optimization Research Center Technical Report TR-2004-01, Columbia University. (**Network, assortment planning**)
- Karaesmen, I. and van Ryzin, G. (2004), "Overbooking with substitutable inventory classes", Operations Research 52(1), 83-104. (**Network, overbooking**)
- van Ryzin, G. and Vulcano, G. (2008), "Computing virtual nesting controls for network revenue management under customer choice behavior", *Manufacturing & Service Operations Management* 10(3), 448-467. (**Network, assortment planning**)
- Zhang, D. and Cooper, W. L. (2005), "Revenue management for parallel flights with customer choice behavior", Operations Research 53(3), 415-431. (**Network, assortment planning**)
- Adelman, D. (2007), "Dynamic bid-prices in revenue management", *Operations Research* 55(4), 647-661. (**Network, capacity allocation**)
- Liu, Q. and van Ryzin, G. (2008), "On the choice-based linear programming model for network revenue management", Manufacturing & Service Operations Management 10(2), 288-310. (**Network, assortment planning**)
- Kunnumkal, S. and Topaloglu, H. (2008), "A refined deterministic linear program for the network revenue management problem with customer choice behavior", Naval Research Logistics Quarterly 55(6), 563-580. (**Network, assortment planning**)
- Topaloglu, H. (2009), "Using Lagrangian relaxation to compute capacity-dependent bid-prices in network revenue management", *Operations Research* 57(3), 637-649. (**Network, capacity allocation**)
- Zhang, D. and Adelman, D. (2009), "An approximate dynamic programming approach to network revenue management with customer choice", Transportation Science 42(3), 381-394. (**Network, assortment planning**)