A TWO-STAGE PROGRAM FOR OPTIMAL GRID POWER BALANCE UNDER UNCERTAINTY

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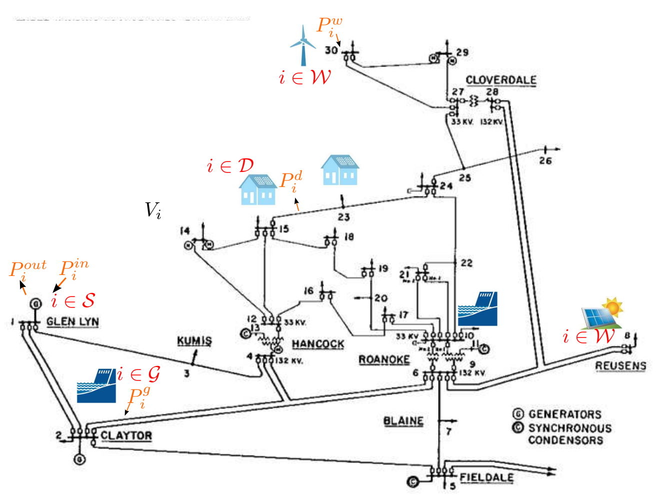
ABSTRACT

We propose a two-stage non-linear stochastic formulation for the *economic dispatch* problem under renewable-generation uncertainty. Certain generation decisions are made only in the first stage and fixed for the subsequent (second) stage, where the actual renewable generation is realized. The uncertainty in renewable output is captured by a finite number of scenarios. Any resulting supply-demand mis-match must then be alleviated using high marginal-cost power sources that can be tapped in short order. We propose two outer approximation algorithms to solve this nonconvex optimization problem to optimality. We show that under certain conditions the sequence of optimal solutions obtained under both alternatives has a limit point that is a globally-optimal solution to the original two-stage nonconvex program. Numerical experiments validate the efficiency and usability of this method of large practical instances.

1 Introduction

Power Balance In The Electrical Grid:



Each node i in the grid is set at a complex-valued voltage V_i, which define the net power flow $(PF_i + \mathrm{i}QF_i)$ into node i governed by Kirchoff's Law on power balance. For real power:

$$PF_i(\mathbf{V}) = \begin{cases} P_i^{in} - P_i^{out} & \forall \quad i \in \mathcal{S} : \text{Nodes with Spot Market Contact} \\ P_i^g & \forall \quad i \in \mathcal{G} : \text{Nodes with Conventional Generators} \\ P_i^w & \forall \quad i \in \mathcal{W} : \text{Nodes with Wind Generators} \\ -P_i^d & \forall \quad i \in \mathcal{D} : \text{Nodes with Energy Load (houses, retail etc.)} \end{cases} \qquad (1)$$

and similarly for the imaginary part $QF_i(\mathbf{V})$. Both terms are non-convex quadratic. Certain reliability/operational constriants are also imposed; for example:

$$\begin{array}{lll} P_i^{min} \le & P_i^g & \le P_i^{max} \qquad \forall \quad i \in \mathcal{G} \\ V_i^{min} \le & |V_i| & \le V_i^{max} \qquad \forall \quad i \in \mathcal{B} \end{array} \qquad (2)$$

The Classical Economic Dispatch Problem:

(let PQ be short-hand for (P,Q))

$$\begin{array}{lll} \min & f(PQ^g) \triangleq & \sum_{i\in\mathcal{G}}(c_{0i}(P_i^g)^2 + c_{1i}P_i^g + c_{2i}) \\ \text{subj. to} & \text{first stage} \quad PQ^g,\ V_i & \text{satisfying (1) and (2).} \end{array} \qquad (\boldsymbol{ED})$$

★ Lavaei and Low (2011) gives a polynomial-time procedure to determine if *quadratic-cost* economic dispatch problem has *zero duality gap*, that is, whether a **global optimum** can be recovered from the dual problem.

- If one exists, procedure constructs the global optimal solution
- Vast majority of real-world problems satisfy this condition

★ We show in Theorem 2 that a zero-duality gap check can be constructed for f that is convex but not quadratic in PQ^g

Economic Dispatch With Intermittent Renewable Sources:

- We model two stages of decisions separated by, say, $5 \sim 15$ mins.
- Second stage recourse provided by expensive, fast-response energy sources P^{in}, Q^{in} like spot-market, peaker generators, demand response etc.
- Better models of intermittent renewables in economic dispatch planning crucially needed for integrating tightly and increasing penetration of renewables
- Stochastic formulation lets standard economic dispatch be more risk-aware

Two-Stage Economic Dispatch:

The first stage of the two-stage problem

$$\begin{array}{ll} \min & f(P^g) + \sum_{s\in\mathcal{P}} p^s\ \omega^s(PQ^g) \\ \text{subj. to} & \text{first stage } PQ^g,\ V_i \text{ satisfying constraints (1) and (2).} \end{array} \qquad (\mathbf{M})$$

Second stage cost ω^s for each scenario s is the optimal objective value of (g and h are quadratic):

$$\begin{array}{lll} \min & (g(P^{in,s}) - h(P^{out,s})) & \\ \text{s. t.} & PF(V^s) = P^g, \quad \text{and} \quad QF(V^s) = Q^g & \text{(from constraints (1))} \\ & \text{constraints (1) and (2) with variables} \quad PQ^{in,s}, PQ^{out,s}, PQ^{w,s}. & \end{array} \qquad (\boldsymbol{S^s})$$

★ **Assumption:** *For every $PQ^g \in [P^{min}, P^{max}] \times [Q^{min}, Q^{max}]$, the subproblem ($\boldsymbol{S^s}$) is feasible.*

★ From Lavaei and Low (2011), we also have that for every PQ^g, the optimal solution can be checked for zero duality gap in polynomial time.

★ **Theorem** 1. *If zero-duality holds for all PQ^g, then $\omega^s(PQ^g)$ is convex. Furthermore, if $\hat{\gamma}$ is the Lagrangian multiplier (i.e., dual solution) corresponding to the constraint with variables PQ^g from the subproblem ($\boldsymbol{S^s}$), then $-\hat{\gamma}$ is a subgradient of ω^s at PQ^g.*

2 Algorithms:

★ *Idea:* We solve a sequence of lower-approximations of the master problem (**M**) by successively augmenting a piecewise-linear lower-approximation of ω^s. Specifically, we generate and use a set of sub-gradients $\{\pi^s\}$ of ω^s. The kth iteration solves:

$$\begin{array}{lllll} \min & f(P^g) + \sum p^s\ \eta^s & & & \\ \text{s.t.} & \text{first stage } PQ^g,\ V_i \text{ satisfying constraints (1) and (2)} & & & \\ \text{and } \forall s,k \quad \text{A:} & \eta^s \ \ge & \omega^s(PQ^{g,k}) & + & (\pi^{s,k})^\intercal(PQ^g - PQ^{g,k}), \\ \text{or B:} & \sum_s p^s\ \eta^s \ \ge & \sum_s p^s \omega^s(PQ^{g,k}) & + & \sum_s p^s(\pi^{s,k})^\intercal(PQ^g - PQ^{g,k}). \end{array} \qquad (M^k)$$

OUTER APPROXIMATION ALGORITHMS

1. Set $\eta^{s,1} = 0\ \forall s \in \mathcal{P}$. Solve the following to set $PQ^{g,1}$

   $\min f(P^g)$

   s.t. first stage $PQ^g,\ V_i$ satisfying (1) and (2).

2. For $k = 1, 2, \ldots$
   - (a) For $s = 1, \ldots, |\mathcal{P}|$: Solve the subproblem (S^s) associated with $PQ^{g,k}$ to get $\omega^s(PQ^{g,k})$ and a subgradient $\pi^{s,k}$.
   - (b) Terminate the algorithm if $\sum_s p^s\ \eta^{s,k} = \sum_s p^s\ \omega^s(PQ^{g,k})$.
   - (c) Else, solve the k-th lower-approximation ($\boldsymbol{M^k}$) master problem to obtain $PQ^{g,k+1}$ and $\eta^{s,k+1}$. Use either A or B outer approximation forms.

3 Convergence:

Theorem 2. *Suppose that $f(PQ^g)$ in ($\boldsymbol{ED}$) is convex but not-quadratic in PQ^g, and an oracle $\pi(PQ^g)$ identifies a subgradient of f at PQ^g. Further, $\pi(PQ^g)$ is uniformly bounded, i.e., there exists a constant C such that $\|\pi(PQ^g)\| \le C$ for all PQ^g. Then a sequence of solutions $\{PQ^{g,k}\}_{k=0,\ldots}$ obtained by solving an outer approximation algorithm similar to 1A either terminates by identifying an optimal solution in a finite number of iterations, or $\lim_{k\to\infty} \Psi(PQ^{g,k}) = \Psi^*$, where Ψ^* is the optimal value of the ($\boldsymbol{ED}$).*

Corollary 3. *Suppose the subgradients $\{\pi^{s,k}\}$ in Algorithms 1A and 1B are chosen from uniformly-bounded dual solutions $\hat{\gamma}$ in Theorem 1. Then the Algorithms either reaches an optimal solution in a finite number of iterations, or generates a sequence with an accumulation point that is optimal for* (**M**).

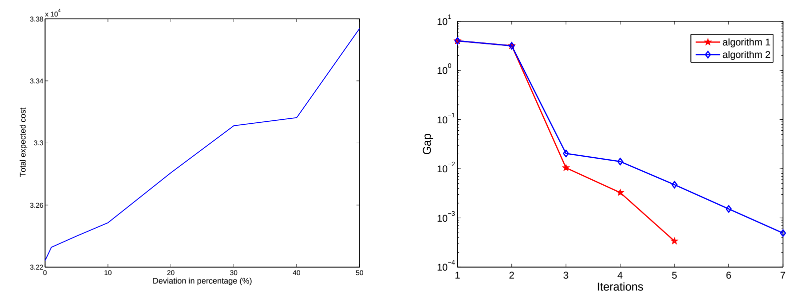
4 Numerical Experiments:

We tested the following standard power systems under the following settings:

- The algorithms terminate when $\sum_{s\in\mathcal{P}} \omega^s(PQ^{g,k+1})p^s - \sum_{s\in\mathcal{P}} \eta^{s,k+1}p^s \le \epsilon = 10^{-3}$.
- Two renewable sources are allowed. Mean power output of each source is sampled uniformly ($\pm 30\%$) around 70% of the average upper limit PQ^{max} of conventional generator capacity.
- Coefficients of import cost $g^s \sim U[1.5, 2.0] \times$ largest coefficients of generation cost f
- Coefficients of export cost $h^s \sim U[0.5, 0.7] \times$ smallest linear coefficients of f.
- Twenty scenarios sampled uniformly with wind generation $\pm 20\%$ from their mean.

Table on performance of the algorithms:

					Algorithm 1A		Algorithm 1B	
Test system	# Buses	# Gen.	Expected cost	Gen. cost	# iter	time	# iter	time
CH9	9	3	2084.9	2053.8	2	0.426	2	0.418
IEEE14	14	5	5872.4	5785.9	2	0.481	2	0.475
IEEE30	30	6	6788.3	6695.7	4	1.706	4	1.657
IEEE57	57	7	37501.4	37331.7	5	0.469	7	0.675
IEEE118	118	54	128321.9	128115.8	4	3.659	4	3.306
IEEE300	300	69	714300.2	714030.3	3	5.048	3	4.932



[**Left**] Effect of varying maximum deviation allowed for renewables on total expected cost for IEEE57.
[**Right**] Errors in each iteration versus the number of iterations for algorithms 1A and 1B for IEEE57.

5 Conclusions

- We propose the first algorithms to solve a two-stage non-convex stochastic formulation for the economic dispatch problem under renewable-generation uncertainty.
- We show that for problem instances that satisfy Lavaei and Low (2011)'s condition, an effective, consistent decomposition scheme can be setup for the two-stage problem. In particular, this facilitates parallel implementations.
- The decomposition scheme solves a sequence of lower-approximations of the first-stage problem. We propose two alternative lower-approximations.
- We show that the sequence of optimal solutions obtained under both alternatives have a limit point that is a globally-optimal solution to the original problem.
- Numerical experiments for a variety of parameter settings show the efficacy of the approach.

References

Lavaei, J., and S. Low. 2011, "Zero Duality Gap in Optimal Power Flow Problem". To appear in *IEEE Transactions on Power Systems.*

Acknowledgments

This material is based upon work supported by the Department of Energy under Award Number DE-OE0000190.