SCHEDULING IN STOCHASTIC JOB SHOPS: HYBRID SIMULATION AND OPTIMIZATION APPROACHES

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Introduction

This work explores hybrid optimization and simulation approaches for scheduling in stochastic job shops. Suppose *m* operations of *p* part types are performed in predefined sequence on *n* machines. The process times are stochastic.

Objective is to assign and sequence these *m x p* operations on *n* machines so as to minimize makespan or maximal completion time.

Attaining a global optimal solution is *NP-Hard*. With hybrid models, we aim to find near optimal solutions within reasonable computational time.

Motivation

Optimization for scheduling is fast and guarantees optimality but parameters are deterministic. Also, for large and complex job shops, computation time is exponential and modelling may be difficult. Also, the solutions may be unrealizable. Typically, MIP formulations such as Manne's[1] model are used for deterministic settings.

Manne's[1] model:

min *Cmax*

subject to

$\sum_{k=1}^{m} r_{imk}(s_{ik}+p_{ik}) \leq Cmax,\ i=1,...,n$

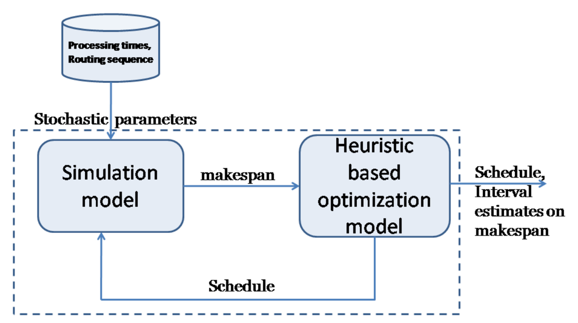
$\sum_{k=1}^{m} r_{imk}(s_{ik}+p_{ik}) - \sum_{k=1}^{m} r_{i,l+1,k}s_{ik} \leq 0,$

$i=1,...,n,\ l=1,...,m-1$

$K(1-x_{ijk}) + s_{jk} - s_{ik} \geq p_{ik},$

$k=1,..,m,\ 1\leq i<j\leq n$

$Kx_{ijk} + s_{ik} - s_{jk} \geq p_{ik},\ k=1,..,m,\ 1\leq i<j\leq n$



Method 1: Simulation-based Optimization

Simulation-based Optimization uses stochastic simulation to evaluate the objective and meta-heuristics to search the solution space. However, they do not guarantee optimum solution, and their convergence is subjective, being poor for small job shops.

Hybrid models combining simulation and optimization approaches are investigated to achieve the following:

- Solutions obtained are realizable (usable) and as close to optimum as possible
- Reduction in computation time
- Incorporation of deterministic and stochastic parameters
- Application to different sizes of job shops

Hybrid Models

Some open and closed loop hybrid models are identified. The simulation models incorporate stochastic parameters.

Klemmt et. al.[2] have proposed the method 3 and discussed it for the semiconductor industry. Simulation-based Optimization is run briefly to get initial solution or bounds on objective functions. These are used with MIP to improve its performance(closer to theoretical optimum in given time).

[1] Manne, A.S.(1960). On the job shop scheduling problem. *Operations Research*. Vol. 8, No. 2, Pages 219-223.

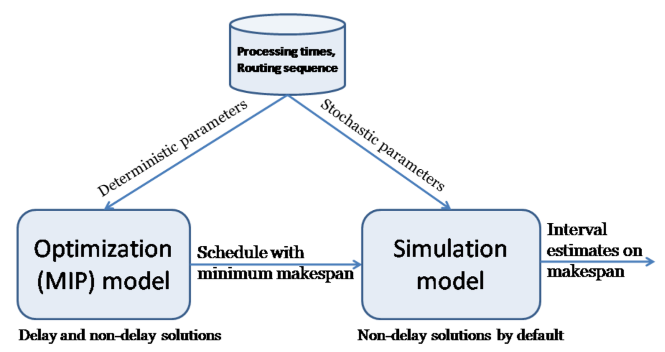
[2] Klemmt, A., Horn, Sven., Weigert, G., Wolter, K. (2009). Simulation-based Optimization vs. mathematical programming: A hybrid approach for optimizing scheduling problems. *Robotics and Computer-Integrated Manufacturing*. Vol. 25, Pages 917-925.

Hybrid Models

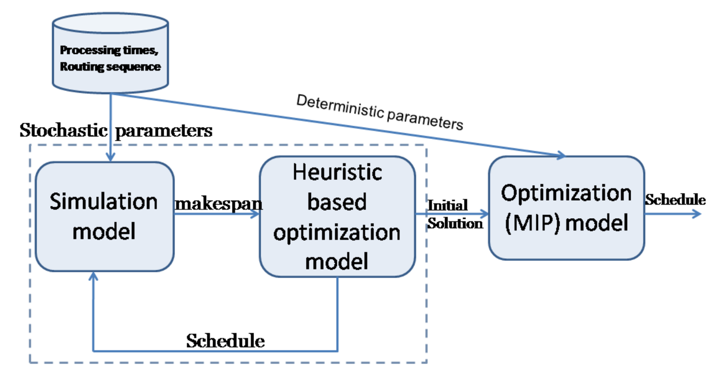
Method 1 Simulation iterated with heuristic based optimization.

Method 2 The MIP model is run with mean processing times (deterministic input). MIP gives a schedule for each machine. MIP searches the solution space for delay as well as non-delay solutions. This schedule is given as input to simulation model along with stochastic processing times. Simulation output helps construct interval estimates on makespan.

Method 3 Simulation-based Optimization is run briefly. A feasible solution is used to warm-start the MIP.



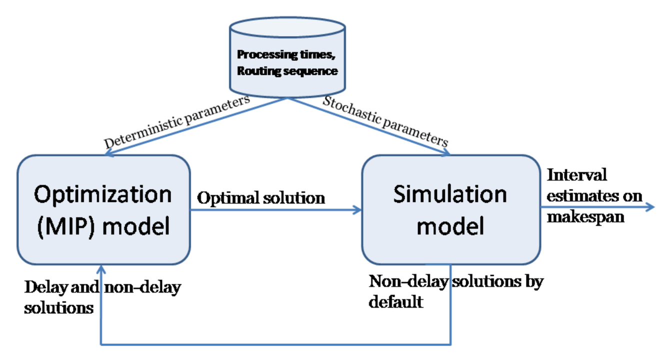
Method 2: Open loop Optimization, Simulation evaluation



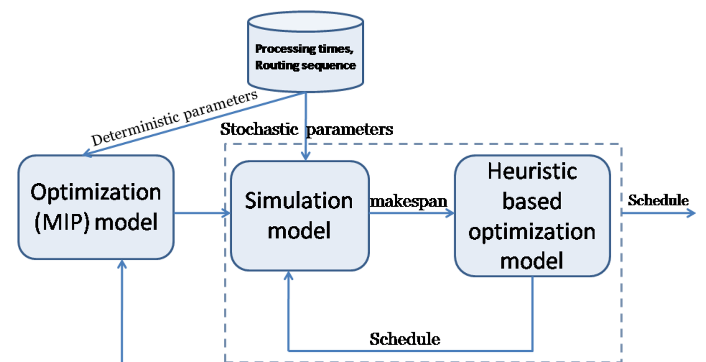
Method 3: Simulation-based Optimization warm-starts Optimization

Method 4 MIP with deterministic inputs generates an optimal solution. The optimum is given to the simulation model to check realize-ability. The difference between the real and *optimum* solutions is used to generate feedback to the MIP.

Method 5 This is a proposed iterative loop between MIP and Simulation-based Optimization.



Method 4: Optimization-Simulation converging iterative loop



Method 5: Optimization to Simulation-based Optimization iterative loop

Experiments and Observations

Benchmark instances from the OR library are tested for comparison.The MIP-formulation by Manne[1] is implemented.

Method 1: Table 1 shows the results of the MIP model (delay) and Method 1 (non-delay):

Table 1: Deterministic settings with no bounds or adjustments

Benchmark	Dimensions	Unknowns	MIP (delay)	Arena (non-delay))
ft06	6x6	90	55 (0.15s)	59 (run 30)
ft08	8x10	280	824 (2.7s)	891 (run 625)
la26	20x10	1900	1266	1315 (run 386)

All optimization models are implemented in CPLEX© 12.1 and simulation models in Arena© 9.0.

Experiments and Observations

Method 2: The 6x6 benchmark instance (ft06) has been modified by using normally distributed processing times.

Table 3: Stochastic settings using Simulation-based Optimization

Dimensions	Makespan	Time	Runs
6x6	283	5min	211

An instance of an optimal solution of the MIP is given as input to stochastic simulation. The following Gantt chart shows a sample solution.

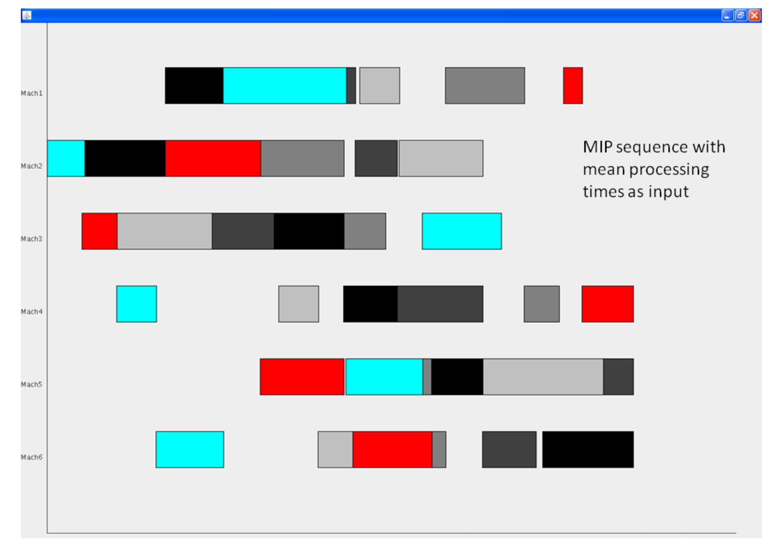


Table 4: Arena with delay and non-delay sequences, 100 replications

Delay/Non-delay	Average	Std. Dev	Half width	min	max
Delay	263	6.46	1.28	246	281
Non-Delay	304	8.72	1.73	281	323

Method 3:The 20x10 benchmark problem (la26) is implemented by using Method 3. By running Simulation-based Optimization for 2 min., a feasible solution is achieved. The makespan of this solution is used as an upper bound on the objective function in the MIP. The bound improves the performance of MIP.

Table 2: Deterministic settings with bounds on objective function in MIP model

Benchmark	Dimensions	Unknowns	Upper bound	MIP (makespan)
la26	20x10	1900	1334	1243

For less than 1000 unknowns, MIP is faster than Simulation-based Optimization and gives theoretical optimum.

For problems with more than 1000 variables, the MIP may not converge to the optimum in restricted time (5 min).

Conclusions and Future work

- Different hybrid frameworks combining optimization and simulation techniques are identified.
- Performance of iterative hybrid methods to be analyzed for their convergence, and computational time
- Integration of hybrid planning and hybrid scheduling models to be explored
- Applicability of the hybrid frameworks for complex systems with stochastic arrival times, planned and unplanned outages, capacitated buffers, shared resources, material handling systems etc are to be investigated

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