Simulation of Revenue Management Systems

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Introduction - PART I

Revenue Management (RM)

The process of understanding and anticipating customer behavior in order to maximize revenue raised from the sale of perishable resources available in limited quantities, such as hotel rooms and airline seats.

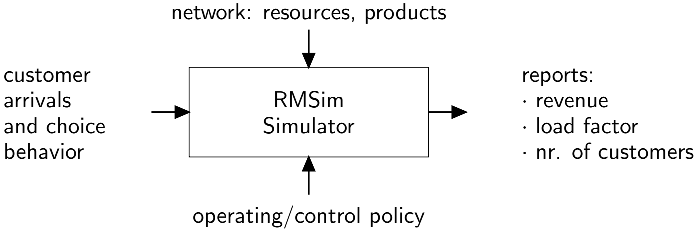
- dynamic programming formulation finds the optimal control policy
  $\Rightarrow$ curse of dimensionality
- mathematical programming models are used to set the control policy
  $\Rightarrow$ oversimplify the actual RM system
- the impact of the full dynamic and stochastic nature of the problem is analyzed via simulation

$\Rightarrow$ library for building simulation programs for revenue management

Components of *RMSim*

Situation:

- perishable resources (expiration date) with limited capacity
- resource is divided in booking classes (purchase conditions)
  - higher fares may be fully or partially refundable
  - lower fare are non-refundable
- offered product is combination of resource-booking class combinations
- customer segmentation
  - customer arrival process
  - customer choice model
- control policy to accept or reject a customers request for a certain product



General Architecture

Design Principles:

- loose coupling: use of interfaces and abstract classes
- high cohesion: classes have well-focused purpose and access to logically related classes

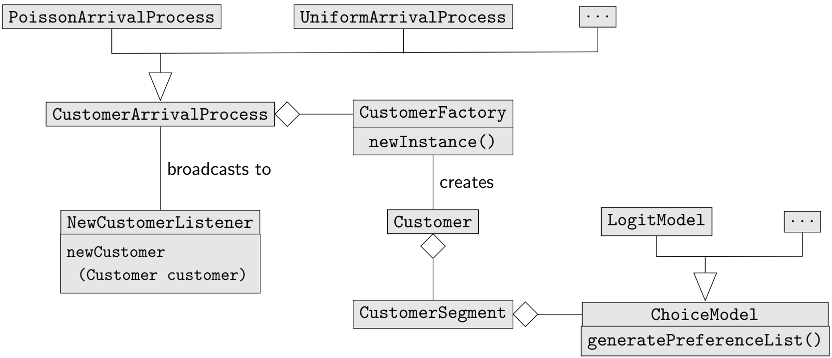
Communication Mechanism:

- observer design pattern: listener objects register to be notified when an event occurs at broadcaster object

Objective:

⇒ flexibility and extensibility to facilitate additional features

Customer Segmentation (1)



Factory design pattern

- `CustomerFactory` interface creates new instances of `Customer` object when `newInstance()` is invoked
- arrival process does not need to know the actual customer segment of the `Customer` object it creates

Customer Segmentation (2)

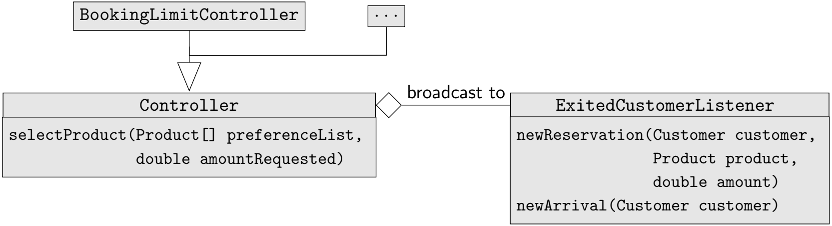
Arrival processes:

- `Poisson`, `PiecewiseConstantPoisson`
- `NormUnif`, `PiecesiseConstantNormUnif`
- `NHPPGammaBeta`
  $\Rightarrow \quad \lambda(t) = D\beta(t)$, where $D \sim$ Gamma and $\beta(t)$ is pdf of Beta

Discrete choice models:

- deterministic
- multinomial logit
- mixed logit
- probabilistic

Control Policy



control policies:

- all-or-nothing vs "smooth"
- booking classes vs virtual classes
- policy type
  - booking limits: partial vs nested (standard, theft)
  - bid prices

XML file - Pre-compiled Simulator (1)

```xml
<rms:rmsystem startingValue="1" xmlns:rms="http://www.iro.umontreal.ca/lecuyer/rmsystem">
   <resource name="AB" capacity="100" expirationValue="4">
      <bookingClass name="HF" bookingLimit="100"></bookingClass>
      <bookingClass name="LF" bookingLimit="71"></bookingClass>
   </resource>
   <resource name="AC" capacity=...

   <product name="HF-AB" price="300">
      <resourceBookingClass resource="AB" bookingClass="HF"></resourceBookingClass>
   </product>
   <product name="LF-ACB" price="130">
      <resourceBookingClass resource="AC" bookingClass="LF"></resourceBookingClass>
      <resourceBookingClass resource="CB" bookingClass="LF"></resourceBookingClass>
   </product>
   ...

   <customerSegment startingValue="1">
      <periodDurations>1</periodDurations>
      <arrivalProcess distributionClass="NORMUNIF">
         <parameters period="1">60 60</parameters>
      </arrivalProcess>
      <choiceModel type="DETERMINISTIC">
         <choiceSet>LF-AB LF-ACB</choiceSet>
      </choiceModel>
   </customerSegment>
   ...

   <controller policy="THEFTNESTEDBOOKINGLIMITS"></controller>
</rms:rmsystem>
```

XML file - Pre-compiled Simulator (2)

Simulation Set-Up

```
<rmsim:repSimParams nrReplications="1000" restrictToPrintedStat="true"
                    xmlns:rmsim="http://www.iro.umontreal.ca/lecuyer/rmsystem">
   <report confidenceLevel="0.95">
      <printedStat measure="NR_CUSTOMERS" perSegment="true"></printedStat>
      <printedStat measure="NR_CUSTOMERS_SATISFIED" perSegment="false"></printedStat>
      <printedStat measure="REVENUE" perSegment="false" perProduct="true"></printedStat>
   </report>
</rmsim:repSimParams>
```

Optimization Set-Up

```
<rmopt:optParams technique="ROD" round="false" nrIterations="30"
                    xmlns:rmopt="http://www.iro.umontreal.ca/lecuyer/rmsystem">
   <parameters name="alpha">0.7</parameters>
   <parameters name="bound">1.0 1.0 1.0 1.0 1.0 1.0</parameters>
</rmopt:optParams>
```

Introduction - PART II

Control policy: bid-price control

- there are threshold values (called bid prices) associated with each resource
- accept a request for product if the price exceeds sum of the bid prices on the resources involved

Popular approach to set the bid prices $\Rightarrow$ Mathematical Programming

Notation

- m resources indexed by i, each with initial capacity c_i units
- n products indexed by j, each with revenue r_j when one unit of product j is sold
- incidence matrix $\mathbf{A} = [a_{ij}] \in \{0,1\}^{m \times n}$:

$$a_{ij} = \begin{cases} 1 & \text{if product } j \text{ requires a unit of resource } i \\ 0 & \text{otherwise} \end{cases}$$

  jth column vector $\mathbf{A}_j$ indicates the subset of resources used by product j
  ith row vector $\mathbf{A}^i$ indicates the subset of products that consume resource i
- state of network is $\mathbf{x} = (x_1, \ldots, x_m)$ and indicates available resource capacities

Deterministic Linear Program (DLP)

$\Rightarrow$ primal solution: allocation

$$V_t^{DLP}(\mathbf{x}) = \begin{array}{ll} \max & \mathbf{r}^\top \cdot \mathbf{y} \\ \text{s.t.} & \mathbf{A} \cdot \mathbf{y} \leq \mathbf{x} \\ & \mathbf{0} \leq \mathbf{y} \leq \sum_{d=t}^{T} \mathbf{D}_d . \end{array}$$

$\Rightarrow$ dual solution: bid prices

Characteristics of DLP

Advantages

- insightful and simple
- easy to implement
- computational efficient

Disadvantages

- deterministic demand
- no distributional information on demand

Time-Dependent Bid Prices - Literature

- Adelman (2007):
  considers the dynamic programming (DP) formulation of the network RM problem and assumes an affine functional form for the value function to formulate a new LP representation of the DP formulation
- Topaloglu (2009), Kunnumkal and Topaloglu (2010):
  relax the capacity constraints in the DP formulation by associating Lagrange multipliers with them $\rightarrow$ decompose the network problem and then compute time-dependent bid prices by focusing on one resource at a time.

Disadvantages

$\Rightarrow$ discretize the booking horizon such that at most one customer arrival within a time period

$\Rightarrow$ substantial overhead in terms of computation and implementation

New Approach to Time-Dependent Bid Prices (1)

Main variables

- assignment variables:

$$z_{j,\tau} = \begin{cases} 1 & \text{if product } j \text{ is available in period } \tau \in [t, T] \\ 0 & \text{otherwise.} \end{cases}$$

- slack variables:

$$\begin{aligned} u_{j,\tau} &= r_j - \mathbf{A}_j^\top \cdot \boldsymbol{\pi}_\tau & \text{if } z_{j,\tau} = 1 \\ v_{j,\tau} &= \mathbf{A}_j^\top \cdot \boldsymbol{\pi}_\tau - r_j & \text{if } z_{j,\tau} = 0 \end{aligned}$$

New Approach to Time-Dependent Bid Prices (2)

$$
\begin{aligned}
V_t^{TDB}(\mathbf{x}) = \quad \max \quad & \mathbf{r}^\top \cdot \sum_{\tau=t}^{T} \mathbf{Y}_\tau \\
\text{s.t.} \quad & \mathbf{A} \cdot \sum_{\tau=t}^{T} \mathbf{Y}_\tau \leq \mathbf{x} \\
& \mathbf{Y}_\tau = \mathbf{D}_\tau \cdot (\mathbf{Z}_\tau)^\top && t \leq \tau \leq T \\
& \mathbf{r} = \mathbf{A}^\top \cdot \boldsymbol{\pi}_\tau + \mathbf{U}_\tau - \mathbf{V}_\tau && t \leq \tau \leq T \\
& \mathbf{U}_\tau \leq K \cdot \mathbf{Z}_\tau && t \leq \tau \leq T \\
& \mathbf{V}_\tau \leq K \cdot (\mathbf{1} - \mathbf{Z}_\tau) && t \leq \tau \leq T \\
& \mathbf{Z}_\tau + \mathbf{V}_\tau \geq \boldsymbol{\epsilon} && t \leq \tau \leq T \\
& \mathbf{U}_\tau, \mathbf{V}_\tau \geq \mathbf{0} && t \leq \tau \leq T \\
& \mathbf{Y} \in \mathbb{R}^{n \times (T-t+1)}, \mathbf{Z} \in \{0,1\}^{n \times (T-t+1)}
\end{aligned}
$$

Incorporate Stochastic Demand - 2 Approaches

Simulation-based Approach to Stochastic Programming

- use demand samples as scenarios
- find one set of robust, time-dependent bid prices that performs well for all demand scenarios
- 2 approaches:
  - include virtual overbookings: when the product is open based on the bid prices, but not enough resources are available $\Rightarrow$ reject the customer request
  - restrict actual overbookings: if the product is open based on the bid prices, assign all demand

Stochastic Demand - Virtual Overbookings (1)

Extra variables

- virtual overbooking variables:

$$w_{j,\tau}^{(s)} = \text{the number of virtual overbookings on product } j \text{ in period } \tau \text{ of scenario } s.$$

- timing of virtual overbooking variables:

$$\overline{w}_{i,\tau}^{(s)} = \begin{cases} 1 & \text{if } (\mathbf{A}^i)^\top \cdot \sum_{\tau \leq t} Y_\tau^{(s)} = x_i, \\ 0 & \text{otherwise.} \end{cases}$$

$\Rightarrow$ used to enforce $y_{j,\tau}^{(s)} = 0$ when $\overline{w}_{i,\tau}^{(s)} = 1$ for some $i \in \mathbf{A}_j$

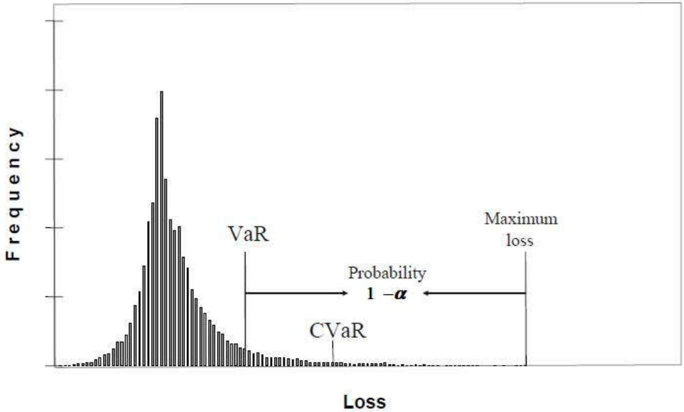
Stochastic Demand - Virtual Overbookings (2)

$$
\begin{aligned}
V_t^{VOD}(\mathbf{x}) = \quad & \max \quad \mathbf{r}^\top \cdot \sum_{s=1}^{S} \sum_{\tau=t}^{T} \mathbf{Y}_\tau^{(s)} & \\
& \text{s.t.} \quad \mathbf{A} \cdot \sum_{\tau=t}^{T} \mathbf{Y}_\tau^{(s)} \leq \mathbf{x} & 1 \leq s \leq S \\
& \mathbf{Y}_\tau^{(s)} + \mathbf{W}_\tau^{(s)} = \mathbf{D}_\tau^{(s)} \cdot (\mathbf{Z}_\tau)^\top & t \leq \tau \leq T, 1 \leq s \leq S \\
& \mathbf{W}_\tau^{(s)} \leq L \cdot \mathbf{A}^\top \cdot \overline{\mathbf{W}}_\tau^{(s)} & t \leq \tau \leq T, 1 \leq s \leq S \\
& \overline{\mathbf{W}}_{\tau-1}^{(s)} \leq \overline{\mathbf{W}}_\tau^{(s)} & t < \tau \leq T, 1 \leq s \leq S \\
& \mathbf{A} \cdot \mathbf{Y}_\tau^{(s)} \leq L(\mathbf{1} - \overline{\mathbf{W}}_{\tau-1}^{(s)}) & t < \tau \leq T, 1 \leq s \leq S \\
& \mathbf{r} = \mathbf{A}^\top \cdot \boldsymbol{\pi}_\tau + \mathbf{U}_\tau - \mathbf{V}_\tau & t \leq \tau \leq T \\
& \ldots \text{ set } \mathbf{U}_\tau, \mathbf{V}_\tau, \mathbf{Z}_\tau &
\end{aligned}
$$

Stochastic Demand - Restrict Overbookings (1)

Portfolio Management:

- value-at-risk measures risk under uncertainty
  - $\longrightarrow$ maximum loss with some confidence level
- conditional value-at-risk quantifies the risk encountered in tail
  - $\longrightarrow$ expected losses strictly exceeding VaR

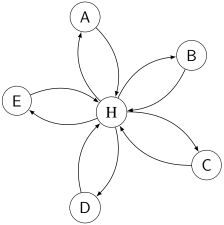


Stochastic Demand - Restrict Overbookings (2)

$$
\begin{aligned}
V_t^{ROD}(\mathbf{x}) = \quad \max \quad & \mathbf{r}^\top \cdot \sum_{s=1}^{S} \sum_{\tau=t}^{T} \mathbf{Y}_\tau^{(s)} \\
\text{s.t.} \quad & \mathbf{A} \cdot \sum_{\tau=t}^{T} \mathbf{Y}_\tau^{(s)} - \boldsymbol{\eta}^{(s)} - \mathbf{w} \leq \mathbf{x} & 1 \leq s \leq S \\
& \mathbf{w} + \frac{1}{1-\alpha} \cdot \frac{1}{S} \sum_{s=1}^{S} \boldsymbol{\eta}^{(s)} \leq \mathbf{b} \\
& \mathbf{Y}_\tau^{(s)} = \mathbf{D}_\tau^{(s)} \cdot (\mathbf{Z}_\tau)^\top & t \leq \tau \leq T, 1 \leq s \leq S \\
& \mathbf{r} = \mathbf{A}^\top \cdot \boldsymbol{\pi}_\tau + \mathbf{U}_\tau - \mathbf{V}_\tau & t \leq \tau \leq T \\
& \text{... set } \mathbf{U}_\tau, \mathbf{V}_\tau, \mathbf{Z}_\tau
\end{aligned}
$$

Numerical Results - Setting

- single hub-and-spoke network with L nodes and 1 hub
- each resource has same capacity
- each resouce has two fare classes: low fare (LF) and high fare (HF)
- price LF $\sim$ unif[15;49]
- price HF product $=$ 5 $\times$ LF product
- total demand LF $=$ 3 $\times$ total demand HF
- demand LF decreases over time, demand HF increases over time



Numerical Results - Approach 1 (VOD)

L	C	T	load factor	DLP RE	LRD		VOD				
						$T = 5$	$S = 5$	$S = 10$	$S = 15$	$S = 20$	$S = 10$, RE
3	1	20	2.59	2.64%	25.72%	23.76%	-25.56%	-11.53%	-7.32%	-3.22%	-3.84%
	3	50	1.75	7.72%	18.81%	19.01%	2.81%	10.93%	13.63%	15.24%	12.49%
	5	100	1.79	12.29%	21.42%	2.45%	12.92%	17.02%	18.44%	19.28%	18.29%
	9	200	1.71	13.29%	8.66%	1.77%	16.03%	18.00%	19.44%	19.84%	20.09%
	19	500	1.68	16.18%	9.20%	5.95%	18.07%	20.47%	21.33%	21.88%	21.36%
5	1	20	1.73	3.36%	26.07%	25.81%	-47.48%	-24.15%	-12.22%	-4.88%	-3.99%
	2	50	1.75	0.45%	18.09%	15.46%	-22.08%	-2.39%	3.63%	5.20%	3.36%
	3	100	1.99	-2.33%	14.33%	2.09%	-7.95%	1.79%	5.07%	5.92%	2.91%
	6	200	1.71	4.15%	10.95%	2.04%	4.57%	7.47%	8.67%	9.78%	8.63%
	13	500	1.64	6.48%	3.67%	-8.32%	6.12%	7.61%	8.21%	9.14%	9.50%

Numerical Results - Approach 2 (ROD)

L	C	T	load factor	$S=10$, $\alpha=0.5$	$S=10$, $\alpha=0.7$	$S=20$, $\alpha=0.5$	$S=20$, $\alpha=0.7$	$S=30$, $\alpha=0.5$	$S=30$, $\alpha=0.7$
3	1	20	2.59	-3.22%	-6.27%	1.20%	1.83%	2.24%	5.40%
	3	50	1.75	13.31%	12.31%	14.87%	16.34%	15.78%	17.53%
	5	100	1.79	18.10%	18.07%	20.08%	19.64%	21.54%	21.24%
	9	200	1.71	19.65%	18.93%	20.88%	20.30%	21.36%	20.77%
	19	500	1.68	21.66%	21.84%	22.34%	22.79%	22.51%	23.41%
5	1	20	1.73	-7.19%	-9.63%	4.06%	5.26%	5.88%	9.34%
	2	50	1.75	3.61%	1.82%	7.49%	8.21%	7.62%	10.45%
	3	100	1.99	3.99%	3.34%	6.92%	8.35%	7.52%	10.01%
	6	200	1.71	9.22%	8.77%	10.46%	11.23%	13.06%	14.13%
	13	500	1.64	8.21%	9.16%	8.88%	10.02%	9.01%	12.24%