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Modeling and assessing the effect of the non-linear dependence between call types in multi-skill call centers

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Abstract

Efficient management of multi-skill call centers is an important challenge in this complex service system. In such call center, multi-skilled agents can handle multiple call types which increase the complexity of agent group management. In order to improve this task, the objective of our work is to investigate the effect of the call type dependency on multi-skill call center performance. For this purpose, copulas are used rather than correlations to take into account the non-linear dependence structure found in realistic data gathered from a major utility company (Hydro-Québec) call center. Using the copula technique allows us to construct the joint distribution of dependent call type's arrival processes and to incorporate an essential empirical feature of tail dependence. After characterizing this dependence structure, simulation experiments are carried out on a multi-skill call center simulator developed with the ContactCenters Java Library. Results show that the widely used assumption of independent call arrivals across call types can lead to substantial misleading estimation of call center output performances. Furthermore we find that the agents' occupancy ratio of multi-skilled groups is highly sensitive to the call arrivals dependency in the joint upper/lower tails.

Context

Empirical data analysis shows asymmetric dependence structure between call types.

Scarcity of works addressing this dependence between call types.

TAEYOON Kim, PHIL KENKEL, and B. WADE BRORSEN: “Forecasting Hourly Peak Call Volume for a Rural Electric Cooperative Call Center”. Journal of Forecasting (2011).

The work doesn’t address the multi-skilling issue

Method

1. Modeling multivariate distributions of daily arrival count using copulas

2. Generate random samples from the specified joint distribution.

3. Implement the generated samples in multi-skill call center simulator to analyze the effect of call type dependency on Output measure of performances

4. Formulate Payout function to quantify the sensitivity of the call center performances to the call type dependence structure

Copulas

A copula *C* is a joint distribution function of standard uniform random variables.

Let $(U_1, \ldots, U_n)^T$ a random vector, where each margin U_i, $i=1,\ldots,n$, is a uniform random variable. C is the joint CDF of $(U_1, \ldots, U_n)^T$

$$C(u_1, \ldots, u_n) = \Pr\{U_1 \leq u_1, \ldots, U_n \leq u_n\} \qquad \text{(Eq.1)}$$

where $U_i \sim U(0,1)$ for $i = 1, \ldots, n$

Copulas

Theorem (Sklar, 1959) : Any multivariate distribution F can be represented with a Copula C.

F : *n*-dimensional distribution function

F_1 ,..., F_n : marginals

$$F(x_1, \cdots, x_n) = C(F_1(x_1), \cdots, F_n(x_n)) \quad \text{(Eq.2)}$$

Multivariate distribution can be separated in:

- Univariate margins
- Multivariate dependence represented by a Copula

Proof: [Sklar, (1959); Nelson, (1999)]

Copulas

Embrechts (1999) pitfalls of Pearson's linear correlation as dependence measure:

- A scalar unable to capture the whole dependence structure
- A correlation of zero does not imply independence
- Vary under increasing transformation

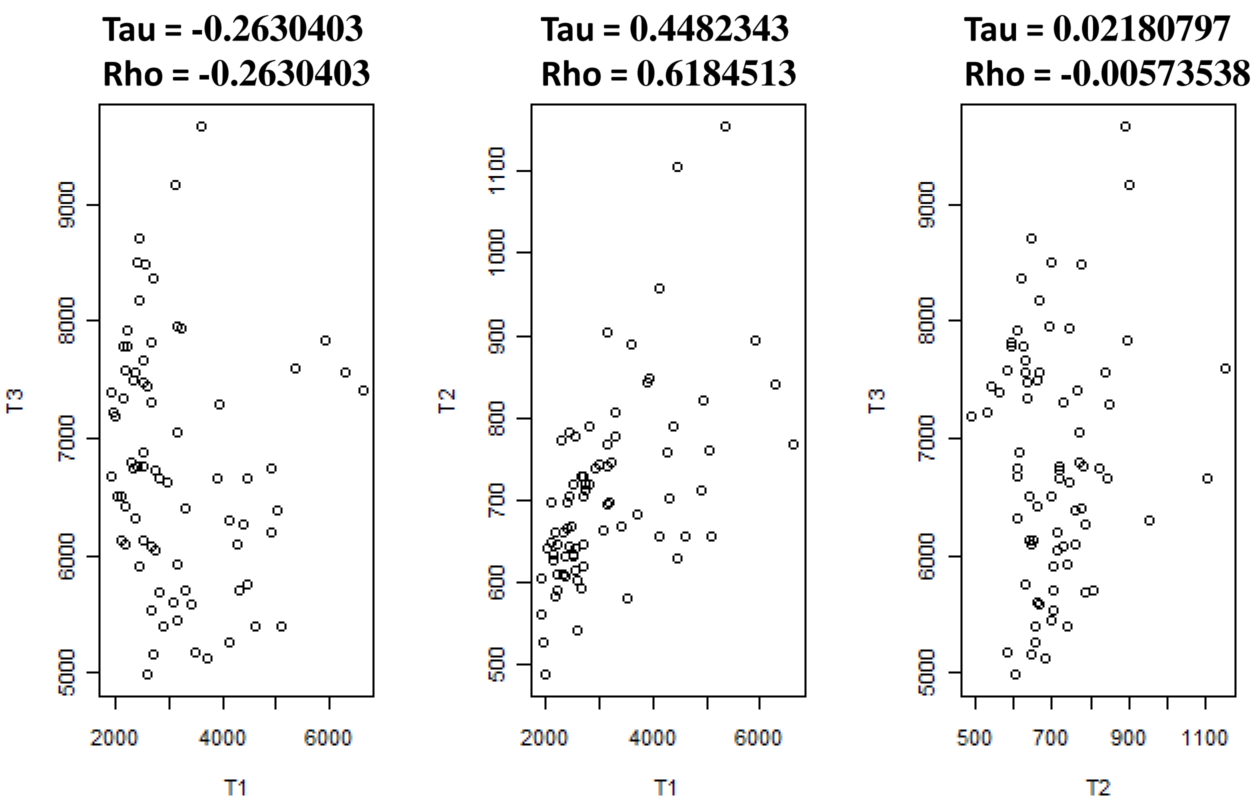
Conclusion on the dependence structure solely based on this correlation measure can lead to severe misinterpretations.

Copula families

Copula (bivariate)	Function	Lower tail λ_L lim $q \to 0$	Upper tail λ_u lim $q \to 1$
$C_{\rho}^{Ga}(u,v)$	$\frac{1}{2\pi\sqrt{1-\rho^2}}\int_{-\infty}^{\Phi^{-1}(u)}\int_{-\infty}^{\Phi^{-1}(v)} e^{-[s^2-2\rho st+t^2]/2(1-\rho^2)}\,dsdt$	0	0
$C_{\rho,v}^{t}(u,v)$	$\frac{1}{2\pi\sqrt{1-\rho^2}}\int_{-\infty}^{\Phi_v^{-1}(u)}\int_{-\infty}^{\Phi_v^{-1}(v)} (1+((s^2+t^2-2\rho st)/(v(1-\rho^2))))^{\frac{-(v+2)}{2}}\,dsdt$	∃	∃
$C_{\theta}^{Gu}(u,v)$	$e^{-[(-\ln u)^{\theta}+(-\ln v)^{\theta}]^{1/\theta}}$	0	∃
$C_{\theta}^{Cl}(u,v)$	$\max[(u^{-\theta}+v^{-\theta}-1)^{-1/\theta},0]$	∃	0
$C_{\theta}^{Fr}(u,v)$	$-\frac{1}{\theta}\ln\left(1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right)$	0	0

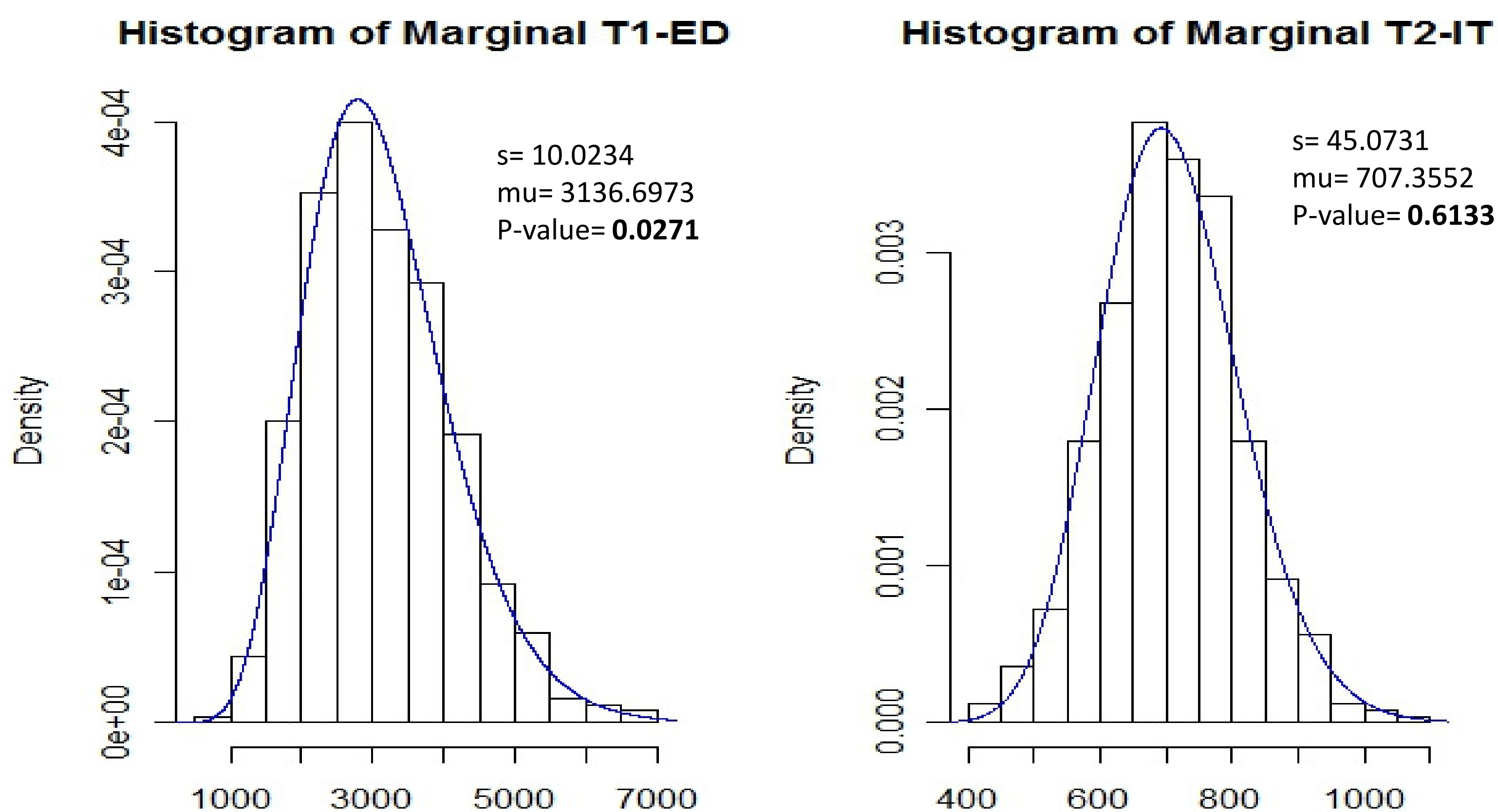
Data Analysis

The daily arrival count corresponding to three main call types T1,T2 and T3 is gathered from 01/05/2010 to 31/10/2010.



Marginal's Specification

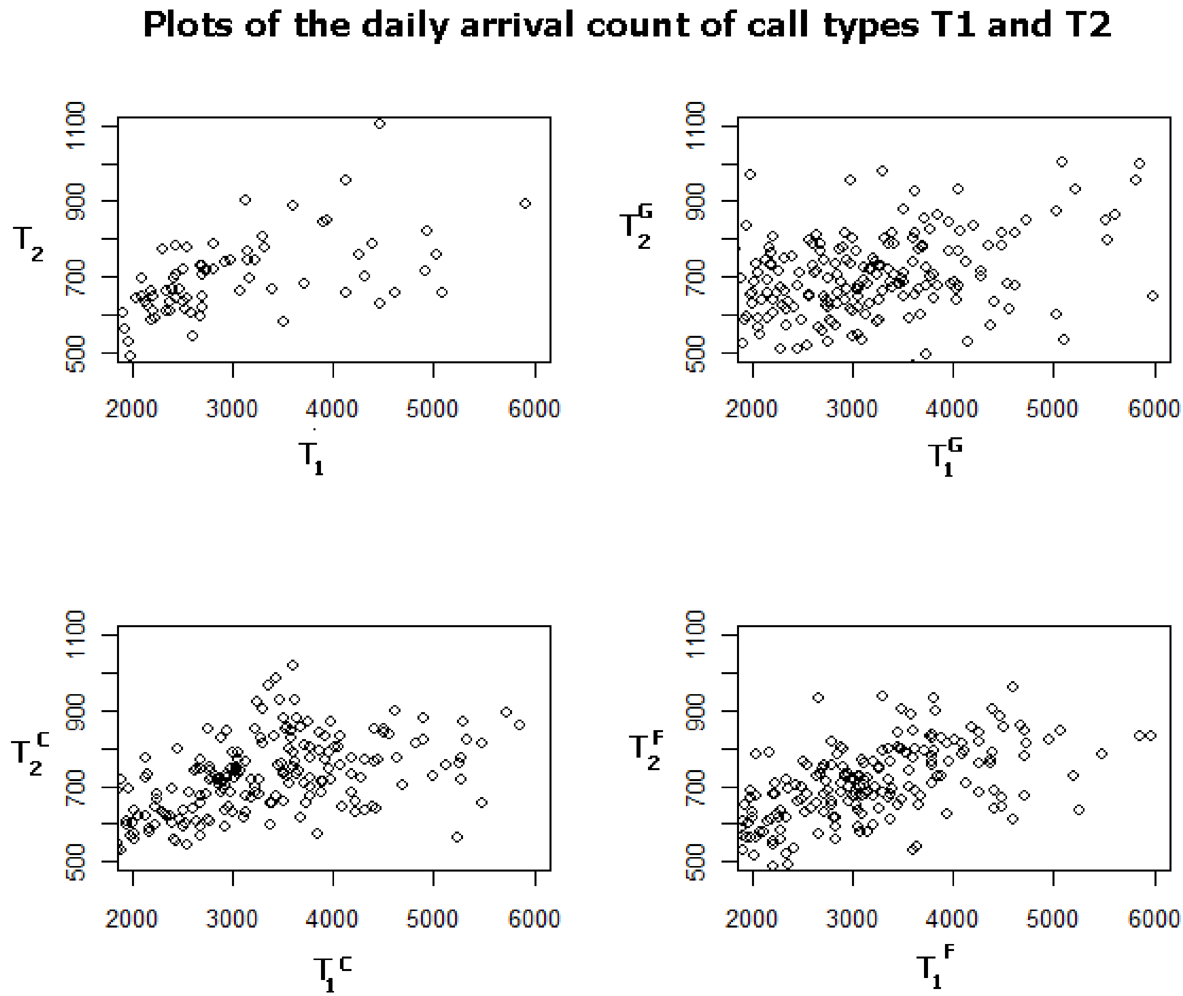
The negative binomial is a good fit to the empirical marginal.



Copula Selection

Copula (bivariate)	Estimates	Maximized log-likelihood	AIC	BIC
$C_{\rho}^{Ga}(u,v)$	0.600	17.0064267	-32.0128534	-29.6821201
$C_{\rho,v}^{t}(u,v)$	0.601	16.3437217	-28.6874435	-28.3567101
$C_{\theta}^{Gu}(u,v)$	1.3702	8.91275837	-15.8255167	-13.4947834
$C_{\theta}^{Cl}(u,v)$	1.7176	**20.3027841**	**-38.6055682**	**-36.2748348**
$C_{\theta}^{Fr}(u,v)$	4.8159	17.3308643	-32.6617286	-30.3309953

Copula Selection



Multi-skill call center Simulator

A multi-skill call center simulator is developed with ContactCenters Java Library.

The tested instance is composed of the 2 call types T1 and T2 and 2 agent group G1 and G2 both have skills to respond to T1 and T2. According to the following agent-skill matrix:

$$A_{2\times 2} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

The service time is gamma distributed for each type, the mean service time for call is computed from realistic statistics gathered from Hydro-Québec. (T1-536s ; T2-579s).

Simulation study

3 Models are tested to compare estimated outputs from the copula-based models with the independent model

- M0- assumes independence across call type arrival process- two univariate negative binomial models
- M1- the bivariate count data model based on the fitted Clayton Copula
- M2- the bivariate count data model based on the fitted Gauss Copula

Simulation study

- The service level estimator

$$g = \mathbb{E}[S_G]/\mathbb{E}[S + A]$$

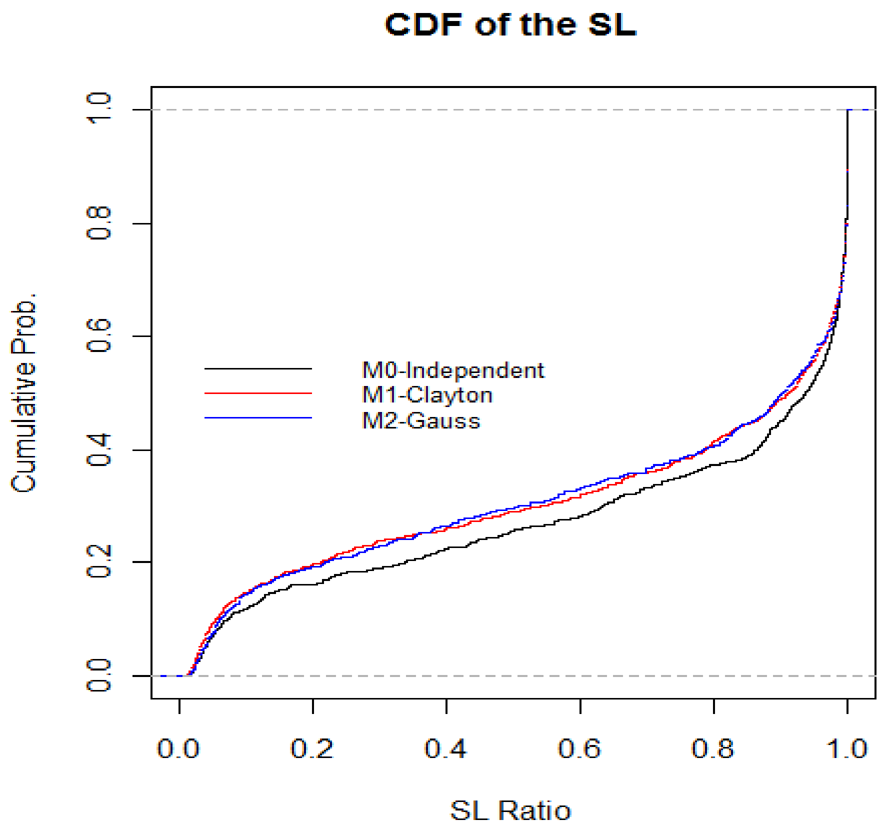
S_G is the number of served contacts having wait less than the acceptable waiting time s, S is the total number of served contacts and A is the number of abandonments after a waiting time s.

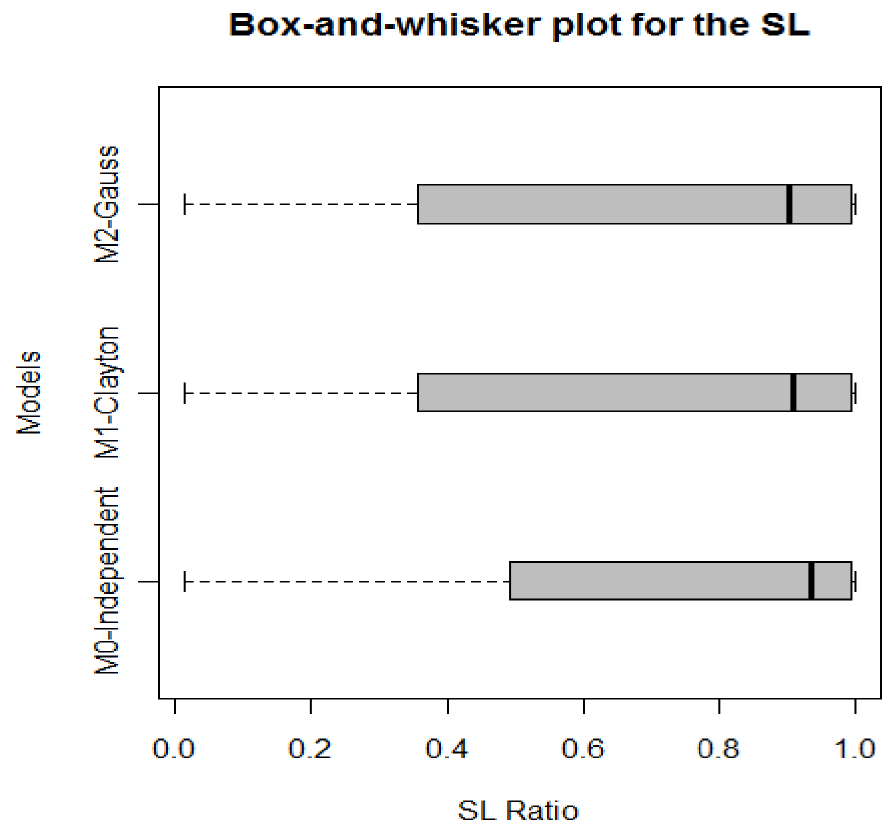
- The Occupancy ratio

$$h = \mathbb{E}\left[\int_0^T N_B(t)\,dt\right]/\mathbb{E}\left[\int_0^T N(t)\,dt\right]$$

N_B is the number of Busy Agent and N is the total number of agents.

Simulation results

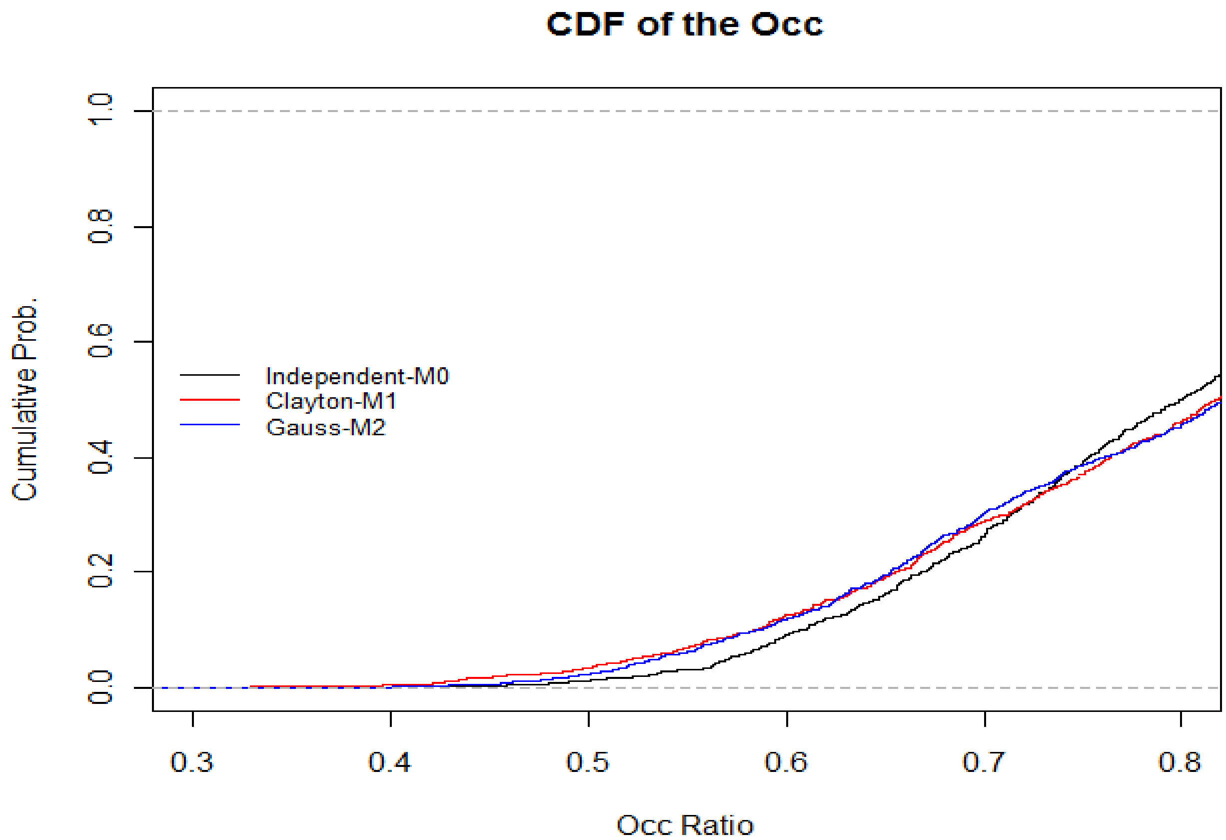




The Service Level CDF from the independent model shows different behavior from the M1 and M2.

The boxplot shows higher inter quartile ranges for model M1 and M2 comparatively to the lower dispersion under the M0 independent model.

Simulation results



The asymmetric left tail dependence induces higher probability of poor Occupancy ratio due to the joint occurrence of low volume of call type 1 and call type 2.

Risk of Service Level Shortfall

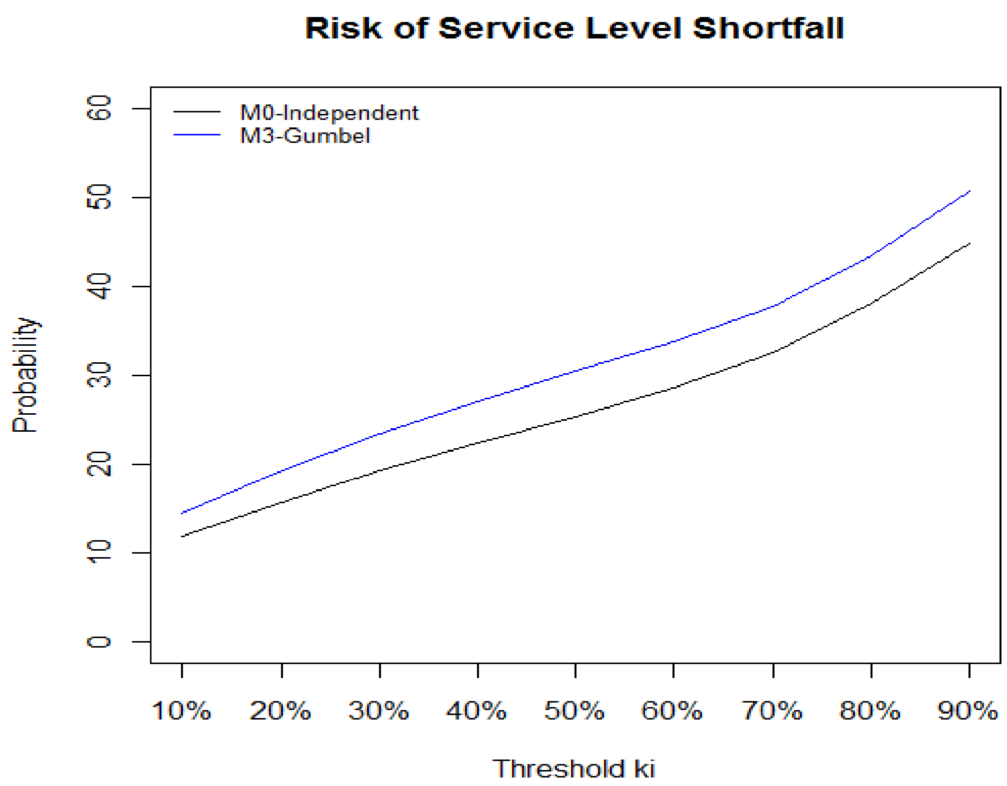
A managerial decision is how much risk of missing the service level target they are willing to tolerate. Conversely, they decide how much insurance to buy in the form of excess capacity.

We study the sensitivity of this risk to the call type right tail dependence structure .

$$\Psi_1(\mathrm{X}) = \mathbb{I}[\hat{g}(\mathrm{X}) < k_i]$$

Were k_i is the acceptable threshold to avoid penalty for service level shortfall.

Risk of Service Level Shortfall



The risk of shortfall is sensitive to the dependence between call types

M0 underestimate the risk of Service Level Shortfall

Main findings

- Assuming independence across call types arrival processes in multiskill call centers may lead to substantial over- under estimation of performance measures.

- The risk of missing the service level target is sensitive to the dependence structure across call types.

- Pooling decision in multiskill call center must take the call type dependence structure into account:

  - to avoid overstaffing it is better to cross train agents to handle less left tail dependent call types.

  - to alleviate the risk of Service Level Shortfall it is better to cross-train agents to handle less right tail dependent call types

Some References

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