A. JAMES CLARK
SCHOOL OF ENGINEERING

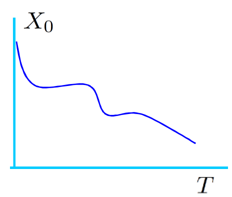
Department of Electrical and Computer Engineering, University of Maryland

Joint Optimization of Pricing Strategies and Inventory Control with Continuous Stochastic Demand

Yongqiang Wang, Michael C. Fu, and Steven I. Marcus

Introduction

- Dynamic pricing and inventory control
  - *Demand depends on the price.*
  - *What amount to order? what price to charge?*
  - *How to jointly optimize the initial order and the pricing policy?*



- Demand model

$$D_t = \int_0^t \lambda(p_s)ds + \int_0^t \sigma(D_s, p_s)dB_s$$

- Inventory level $X_t = X_0 - D_t$

$$dX_t = -dD_t = -\lambda(p_t)dt - \sigma(X_0 - X_t, p_t)dB_t$$

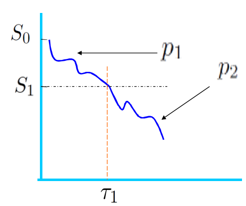
- Maximize the following objective

$$E\left[\int_0^{\tau\wedge T} p_s dD_s - c(X_s)ds\right]$$

Simulation-based Method

- Pricing policy

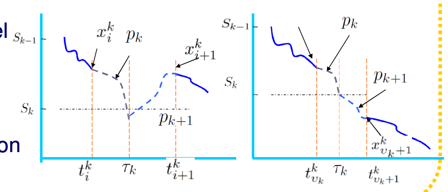
$$S = S_0 > S_1 > \cdots > S_{N-1} > S_N = 0$$
$$\tau_k = \inf\{t \geq 0 : X_t = S_k\}$$
$$\pi_f = \{S_1, \ldots, S_{N-1}, p_1, \ldots, p_N\}$$



- Objective

$$\sup_{\pi_f, S} E\left[\sum_{k=1}^{N} \int_{\tau_{k-1}}^{\tau_k \wedge T} -p_k dX_s - c(X_s)ds \,|\, X_0 = S\right]$$

- Simulation of inventory level
- Derive efficient gradient estimators w.r.t decision variables
- Use stochastic approximation to find optimal value

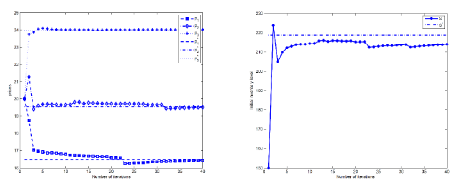


MAIN RESULTS

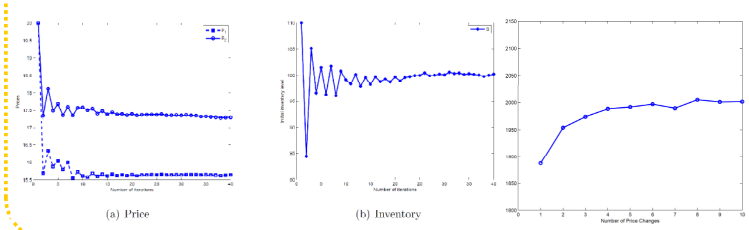
- Developed an efficient simulation algorithm to generate sample paths of inventory level
- Developed an efficient gradient estimator for stopping times based on Brownian bridge
- Derived an unbiased gradient estimator for performance functions with indicator functions
- Proved the convergence of SA with the obtained gradient estimator

Table 1: Infinite Time Sensitivity Estimation Results (Standard Error in Parentheses)

	$\sigma = 1$	$\sigma = 3$	$\sigma = 5$	$\sigma = 7$
$\partial\hat{V}/\partial p_1$				
LR	5.91(2.55)	4.02(1.27)	4.30(0.78)	5.03(1.24)
PW	5.47(0.18)	5.40(0.32)	5.08(0.40)	3.88(0.50)
ANA	5.52	5.26	4.72	3.92
$\partial\hat{V}/\partial p_2$				
LR	18.85(2.00)	16.03(0.51)	15.44(0.38)	14.57(0.32)
PW	16.70(0.09)	16.47(0.17)	16.04(0.22)	15.43(0.27)
ANA	16.63	16.367	15.83	15.03
$\partial\hat{V}/\partial p_3$				
LR	27.75(0.01)	27.47(0.03)	26.97(0.06)	26.12(0.09)
PW	27.73(0.01)	27.47(0.03)	26.99(0.06)	26.36(0.09)
ANA	27.74	27.48	26.94	26.14
$\partial\hat{V}/\partial S$				
LR	10.28(0.32)	9.85(0.14)	9.88(0.08)	9.83(0.13)
PW	10.05(0.02)	10.03(0.04)	9.98(0.04)	9.86(0.05)
ANA	10.00	9.96	9.88	9.76
$\partial\hat{V}/\partial\alpha$				
LR	-505.2(0.3)	-508.2(0.9)	-515.3(1.5)	-531.1(2.2)
PW	-505.3 (0.3)	-509.9(0.9)	-516.0(1.5)	-530.8(2.1)
ANA	-500.5	-504.5	-512.5	-524.5
$\partial\hat{V}/\partial\sigma$				
LR	0.27(1.66)	-1.75(0.90)	-4.46(0.48)	-5.99(0.48)
PW	-1.51(0.31)	-3.28(0.34)	-4.642(0.39)	-7.38(0.44)
ANA	-1.00	-3.00	-5.00	-7.00



(a) Price (b) Inventory



(a) Price (b) Inventory

CONCLUSIONS

- Developed a simulation-based framework for solving dynamic pricing and inventory control problems with very general demand models
- Unbiasedness of the gradient estimator and theoretical convergence of the algorithm
- Effectiveness demonstrated by numerical results.